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Event-triggered attitude control of spacecraft

Baolin Wu [a,*], Qiang Shen [b], Xibin Cao [a]

[a] *Harbin Institute of Technology, 150080 Harbin, People's Republic of China*

[b] *National University of Singapore, Singapore 117582, Singapore*

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Abstract

The problem of spacecraft attitude stabilization control system with limited communication and external disturbances is investigated based on an event-triggered control scheme. In the proposed scheme, information of attitude and control torque only need to be transmitted at some discrete triggered times when a defined measurement error exceeds a state-dependent threshold. The proposed control scheme not only guarantees that spacecraft attitude control errors converge toward a small invariant set containing the origin, but also ensures that there is no accumulation of triggering instants. The performance of the proposed control scheme is demonstrated through numerical simulation.

Keywords: Spacecraft control; Attitude stabilization; Event-triggered control; Limited communication

1. Introduction

The objective of spacecraft attitude control system is to make the spacecraft achieve fine pointing, rapid maneuvering, accurate tracking and other desired performances. Since a spacecraft involves nonlinear and highly coupled dynamics, and may subject to unexpected factors, such as external disturbances, inertia matrix uncertainties and actuators failures/faults, numerous control schemes have been investigated for solving the spacecraft attitude control problem. These include sliding mode control (Boskovic et al., 2001; Liang et al., 2007; Du and Li, 2012; Hu et al., 2013; Zou, 2014; Eshghi and Varatharajoo, 2017), optimal control (Luo et al., 2005; Boyarko et al., 2011; Shen et al., 2015; Jikuya et al., 2016), adaptive control (Ahmed et al., 1998; Cai et al., 2008; Xia et al., 2011), and iterative learning control (Wu et al., 2015), etc.

Plug-and-play spacecraft (Lyke, 2012) have received a significant research attention over the past few years. The functional components of plug-and-play spacecraft are independent modules. These functional modules interact through wireless links. The plug-and-play spacecraft offers more flexibility and robustness than conventional monolithic spacecraft. Data transmission among independent modules is performed by low-cost wireless network in this kind of spacecraft. Data communication rate of low-cost wireless network is limited. As a result, a key problem encountered for plug-and-play spacecraft is how to decrease the signal transmission burden among independent modules. Thus, novel attitude control method with limited communication should be investigated. Although there are plenty of results on spacecraft attitude control in the literature, there is few result available which investigates attitude control with limited communication.

One possible solution to the problem of attitude control with limited communication is to utilize signal quantization with recently proposed quantizer. Based on this idea, Wu (2016) proposes a control scheme for spacecraft attitude

* Corresponding author at: Research Center of Satellite Technology, Harbin Institute of Technology, Room 507, Block B3, No. 2 Yikuang Street, Nangang District, Harbin 150080, People's Republic of China.

E-mail address: wuba0001@e.ntu.edu.sg (B. Wu).

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stabilization with control torque quantized by a logarithmic quantizer, and a guideline to choose the quantizer parameters. Besides signal quantization, event-triggered control is another effective method to reduce the waste of communication and computation resources for network control systems (Tabuada, 2007). The main difference between the classical sample-data control and event-triggered control lies in how the controller is implemented to the system. For the classical control, the control signal is sampled and applied to the system periodically no matter the system needs it or not. While under the event-triggered control strategy, the controller is activated only when a defined measurement error exceeds a given threshold, and only triggered data at some discrete triggered times need to be transmitted. Therefore, the event-triggered method can be expected to have an efficient effect on reducing the communication pressure for spacecraft attitude control system.

Note that the results of event-triggered control for nonlinear systems are still very limited. Event-triggered control algorithms are investigated in Tabuada (2007) and Tallapragada and Chopra (2013) for stabilization of nonlinear systems, and in Postoyan et al. (2015) for trajectory tracking in nonlinear systems. The above event-triggered control algorithms in Tabuada (2007), Tallapragada and Chopra (2013), Postoyan et al. (2015) are based on the input to-state stability (ISS) assumption which implies the existence of a feedback control law ensuring an ISS property with respect to measurement errors. The design of both the controller and the event-triggering condition simultaneously for nonlinear systems has been proposed in Sahoo et al. (2016), Xing et al. (2016), Li et al. (in press). Adaptive neural network-based event-triggered control law is studied in Sahoo et al. (2016) for single-input single-output uncertain nonlinear discrete-time systems. Adaptive backstepping control-based event-triggered control method is presented in Xing et al. (2016) for a class of uncertain nonlinear systems. Furthermore, observer-based fuzzy adaptive event-triggered control law is proposed in Li et al. (in press) for uncertain nonlinear systems. Though some results have been established for event-triggered control for nonlinear systems, to our best of knowledge, there is still no consideration of spacecraft attitude control with an event-triggered mechanism. Since attitude dynamics is nonlinear and highly coupled, it is not straightforward to apply the above event-triggered control laws to attitude control of spacecraft.

This paper aims to provide a solution to the problem of spacecraft attitude stabilization control with limited communication. An event-triggered control method is investigated to reduce the communication pressure. Under the event-triggered strategy, the information of attitude and control torque only need to be transmitted at some discrete triggered times when a defined measurement error exceeds a state-dependent threshold. Through Lyapunov analysis, it is shown the proposed control law ensures that attitude control errors converge toward a small invariant set containing the origin in spite of external disturbances. A positive lower bound on inter-update time is also guaranteed to avoid accumulation of triggering instants. Simulation results demonstrate the efficiency of the proposed event-triggered attitude control law, and data to be sent over the communication channel under the proposed attitude control scheme is greatly reduced.

The remaining parts of this paper are organized as follows. Section 2 introduces the attitude dynamics of spacecraft. Section 3 presents the problem formulation. Section 4 proposes an event-triggered controller design scheme for attitude stabilization, and the stability analysis of the resulting closed-loop system is also given in this section. Section 5 presents the simulation results, while the conclusions are drawn in Section 6.

2. Spacecraft attitude dynamics

With the assumption of rigid body movement, the kinetics of a spacecraft can be established from Euler's moment equation as (Ahmed et al., 1998):

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{u} + \boldsymbol{d} \tag{1}$$

where $\boldsymbol{\omega} = \boldsymbol{\omega}(t) \in \mathbb{R}^3$ denote the body angular velocity of the spacecraft with respect to the inertial frame $\mathcal{I}$, expressed in the body-fixed frame $\mathcal{B}$; $\boldsymbol{J} = \boldsymbol{J}^T \in \mathbb{R}^{3\times 3}$ denotes the positive-definite inertia matrix of the spacecraft. $\boldsymbol{u} = \boldsymbol{u}(t) \in \mathbb{R}^3$ denotes the control torque expressed in the body-fixed frame $\mathcal{B}$; $\boldsymbol{d} \in \mathbb{R}^3$ denotes the external disturbances expressed in the body-fixed frame $\mathcal{B}$; The notation $\boldsymbol{\omega}^{\times}$ for the vector $\boldsymbol{\omega} = [\,\omega_1 \quad \omega_2 \quad \omega_3\,]^T$ is employed to denote the skew-symmetric matrix as below

$$\boldsymbol{\omega}^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

The nonlinear differential equations that govern the kinematics of the rigid spacecraft in terms of quaternion are described as follows (Ahmed et al., 1998):

$$\dot{\boldsymbol{q}}_v = \frac{1}{2}(\boldsymbol{q}_v^{\times} + q_0\mathbf{I}_3)\boldsymbol{\omega} \tag{2}$$

$$\dot{q}_0 = -\frac{1}{2}\boldsymbol{q}_v^T\boldsymbol{\omega} \tag{3}$$

$(\boldsymbol{q}_v, q_0) \in \mathbb{R}^3 \times \mathbb{R}$ denotes the unit quaternion representing the orientation of a body-fixed frame $\mathcal{B}$ with respect to an inertial frame $\mathcal{I}$ and satisfies the constraints $\boldsymbol{q}_v^T\boldsymbol{q}_v + q_0^2 = 1, \boldsymbol{q}_v \in \mathbb{R}^3$ and $q_0 \in \mathbb{R}$ denote the vector and scalar components, respectively; $\mathbf{I}_3$ denotes the 3×3 identity matrix.

Property 1. *The inertia matrix of spacecraft* $\boldsymbol{J}$, *which is symmetric and positive-definite, satisfies the following bounded condition*:

$$J_{\min}\|\boldsymbol{x}\|^2 \leqslant \boldsymbol{x}^T\boldsymbol{J}\boldsymbol{x} \leqslant J_{\max}\|\boldsymbol{x}\|^2, \quad \forall \boldsymbol{x} \in \mathbb{R}^3 \tag{4}$$

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where $J_{\min}$ and $J_{\max}$ are some positive constants, respectively.

To facilitate control system design, the following assumption is used in the subsequent developments.

Assumption 1. The external disturbance $\boldsymbol{d}$ is bounded such that $\|\boldsymbol{d}\| \leqslant d_{\max}$, where $d_{\max}$ is a positive constant.

Remark 1. It is reasonable to assume the external disturbance $\boldsymbol{d}$ is bounded, since all the environmental disturbances due to aerodynamic drag, solar radiation pressure, gravitation, or magnetic forces are bounded in practice.

3. Problem formulation

The objective of this paper is to propose an attitude stabilization control law and event-triggered rule to reduce the information to be transmitted over the communication channel, and ensure that the ultimate attitude tracking error converges towards a residual set in the presence of external disturbances.

The structure of the proposed event-triggered control scheme with communication network between the spacecraft and the attitude control module is shown in Fig. 1. In the proposed strategy, a trigger mechanism is incorporated in the sensor module of the spacecraft to determine the event-triggering instants by evaluating the event-triggering condition in an approximately continuous way. The sampling frequency of the spacecraft sensors, such as star track and gyro is high up to 32 Hz. Thus, evaluating the event-trigger condition in an approximately continuous way can be considered to be feasible. In the event of the violation of the event-trigger condition, the information of attitude $\boldsymbol{s}$ defined in (5) is transmitted from the sensor module of the spacecraft to the attitude control module. Then, the signal of control toque $\boldsymbol{u}$ is updated in the attitude control module and transmitted to the actuators of spacecraft. The zero-order holds (ZOHs) are employed to hold the last transmitted information of attitude and control torque at the attitude control module and the actuator module, respectively, until the next transmission is received.

Let t_i for $i = 0, 1, 2, \ldots$ be triggering instants at which the control torque is computed and updated. The traditional sampled-data attitude control system updates the signal of control torque in a periodic way. It means that $t_{i+1} - t_i = T$, where the sample time $T > 0$ is constant. The execution of control torque in the traditional sampled-data attitude control system can be viewed as being time-triggered. On the contrary, in event-triggered control system, the sampling period denoted by $T_i = t_{i+1} - t_i$ is no longer constant. The sampling period T_i is named as inter-update time. Once the signal of control torque is updated, it is kept constant until the next time instant is triggered. The event-triggering instants, t_i, are determined by comparing the measurement error $\boldsymbol{e}(t)$ for all $t \in \mathbb{R}_+$ to the state dependent threshold. The measurement error $\boldsymbol{e}(t)$ and the state dependent threshold are to be designed in the following section.

4. Event-triggered controller design

In this section, an event-triggered feedback controller is proposed for the attitude control system described in (1)–(3) in the presence of external disturbances to reduce communication pressure.

To design the event-triggered attitude control scheme, the filtered error vector is defined as follows:

$$\boldsymbol{s} = \boldsymbol{\omega} + \beta \boldsymbol{q}_v \tag{5}$$

where $\boldsymbol{q}_v$ is the vector part of quaternion, $\boldsymbol{\omega}$ is the angular velocity, and $\beta > 0$ denotes a constant parameter to be designed.

The event-triggered attitude stabilization control law is proposed as below.

$$\boldsymbol{u}(t) = -k\boldsymbol{s}(t_i), \quad \textit{for } t \in [t_i,\ t_{i+1}), \quad i = 0, 1, 2, \ldots \tag{6}$$

where k denotes a positive constant chosen by designer. The control torque is held constant in inter-update time i.e., $[t_i,\ t_{i+1})$.

Remark 2. The control law in (6) has a very simple structure. The information to be transmitted from the sensor module to the attitude control module is just the filtered error vector $\boldsymbol{s}$. Compared to other control laws in

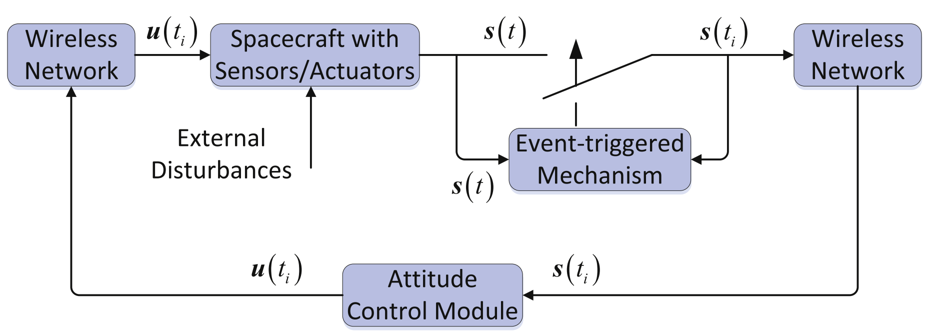

Fig. 1. Structure of the event-triggered attitude control system.

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the literature, the number of variables need to be transmitted by the proposed control law (6) is the minimum.

Define the measurement error as

$$\boldsymbol{e} \triangleq \boldsymbol{s}(t_i) - \boldsymbol{s}, \quad t \in [t_i,\ t_{i+1}) \tag{7}$$

Remark 3. The measurement error $\boldsymbol{e}$ plays a crucial role in the implementation of the event-triggered control scheme. The triggering instant t_{i+1} is decided by monitoring the evolution of $\boldsymbol{e}$ in an approximately continuous way until it crosses a state-dependent threshold. Hence, this strategy is suitable for attitude control of the fractionated spacecraft and the plug-and-play spacecraft where minimum communication is desired.

The following triggering event is considered

$$t_{i+1} = \min\{t \geqslant t_i : \|\boldsymbol{e}\| \geqslant \alpha\|\boldsymbol{s}\| + \gamma\}, \quad i = 0, 1, 2, \ldots \tag{8}$$

where $\alpha \in (0, 1)$, and $\gamma > 0$ is a small parameter that is designed to ensure the controller avoids Zeno behavior.

Remark 4. Larger the value of the parameter α, longer the inter-update times are achieved. As a result, less communication and computation are required. On the other hand, larger α results in larger stead-state attitude control errors. Thus, the parameter α should be designed based on a trade-off between communication burden and stead-state attitude control errors.

It is shown that the resulting closed-loop event-triggered system is uniformly ultimately bounded in the following theorem.

Theorem 1. *Consider the spacecraft attitude stabilization system (1)–(3), the control law (6), and the event-triggering condition (8). If Assumption 1 is satisfied, then the attitude error $\boldsymbol{q}_v$ and angular velocity error $\boldsymbol{\omega}$ are uniformly ultimately bounded, and converge to a small residual set, that is, $\lim_{t\to\infty}\|\boldsymbol{\omega}(t)\| \in \boldsymbol{\Omega}_\omega$ and $\lim_{t\to\infty}\|\boldsymbol{q}_v(t)\| \in \boldsymbol{\Omega}_q$, where $\boldsymbol{\Omega}_\omega$ and $\boldsymbol{\Omega}_q$ are defined as*

$$\begin{aligned} \boldsymbol{\Omega}_\omega &= \left\{\boldsymbol{\omega} \middle| \ \|\boldsymbol{\omega}\| \leqslant \sqrt{\frac{2\varphi}{J_{\min}\varepsilon}}\right\}, \\ \boldsymbol{\Omega}_q &= \left\{\boldsymbol{q}_v \middle| \ \|\boldsymbol{q}_v\| \leqslant \sqrt{\frac{\varphi}{k\beta\varepsilon}}\right\} \end{aligned} \tag{9}$$

where ε and $\varphi$are positive constants defined in (23).

Proof. Choose the candidate of Lyapunov function as

$$V = k\beta(1-q_0)^2 + k\beta\boldsymbol{q}_v^T\boldsymbol{q}_v + \frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{J}\boldsymbol{\omega} \tag{10}$$

Then the time derivative of V along the motion of (1)–(3) is derived as

$$\begin{aligned} \dot{V} &= -2k\beta\dot{q}_0 + \boldsymbol{\omega}^T\boldsymbol{J}\dot{\boldsymbol{\omega}} \\ &= k\beta\boldsymbol{q}_v^T\boldsymbol{\omega} + \boldsymbol{\omega}^T(-\boldsymbol{\omega}^\times\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{u} + \boldsymbol{d}) \end{aligned} \tag{11}$$

Note that

$$\boldsymbol{\omega}^T(-\boldsymbol{\omega}^\times\boldsymbol{J}\boldsymbol{\omega}) = 0 \tag{12}$$

Combining (11) and (12) yields

$$\dot{V} = k\beta\boldsymbol{q}_v^T\boldsymbol{\omega} + \boldsymbol{\omega}^T(\boldsymbol{u} + \boldsymbol{d}) \tag{13}$$

Substituting the control law (6) into (13) leads to

$$\begin{aligned} \dot{V} &= k\beta\boldsymbol{q}_v^{\mathrm{T}}\boldsymbol{\omega} - \boldsymbol{\omega}^{\mathrm{T}}k\boldsymbol{s}(t_i) + \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{d} \\ &= k\beta\boldsymbol{q}_v^{\mathrm{T}}\boldsymbol{\omega} - \boldsymbol{\omega}^{\mathrm{T}}\left(k\boldsymbol{e} + k\boldsymbol{\omega} + k\beta\boldsymbol{q}_v\right) + \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{d} \\ &= -\boldsymbol{\omega}^{\mathrm{T}}k\boldsymbol{e} - \boldsymbol{\omega}^{\mathrm{T}}k\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{d} \\ &\leqslant -k\|\boldsymbol{\omega}\|^2 + k\|\boldsymbol{\omega}\|\|\boldsymbol{e}\| + d_{\max}\|\boldsymbol{\omega}\| \end{aligned} \tag{14}$$

where (7) is used to derive the second equation, and Assumption 1 is employed to obtain the last inequality.

Considering the triggering condition in (8), it follows from (14) that

$$\dot{V} \leqslant -k\|\boldsymbol{\omega}\|^2 + k\|\boldsymbol{\omega}\|(\alpha\|\boldsymbol{s}\| + \gamma) + d_{\max}\|\boldsymbol{\omega}\| \tag{15}$$

Note that

$$\|\boldsymbol{s}\| = \|\boldsymbol{\omega} + \beta\boldsymbol{q}_v\| \leqslant \|\boldsymbol{\omega}\| + \beta\|\boldsymbol{q}_v\| \leqslant \|\boldsymbol{\omega}\| + \beta \tag{16}$$

Substituting (16) into (15) yields

$$\begin{aligned} \dot{V} &\leqslant -k\|\boldsymbol{\omega}\|^2 + k\|\boldsymbol{\omega}\|[\alpha(\|\boldsymbol{\omega}\| + \beta) + \gamma] + d_{\max}\|\boldsymbol{\omega}\| \\ &= -k(1-\alpha)\|\boldsymbol{\omega}\|^2 + (k\gamma + k\alpha\beta + d_{\max})\|\boldsymbol{\omega}\| \end{aligned} \tag{17}$$

Introducing a constant η satisfying $0 < \eta < k(1-\alpha)$, it is obtained

$$\begin{aligned} \dot{V} &\leqslant -[k(1-\alpha)-\eta]\|\boldsymbol{\omega}\|^2 - \eta\|\boldsymbol{\omega}\|^2 + (k\gamma + k\alpha\beta + d_{\max})\|\boldsymbol{\omega}\| \\ &\leqslant -[k(1-\alpha)-\eta]\|\boldsymbol{\omega}\|^2 + \frac{(k\gamma + k\alpha\beta + d_{\max})^2}{4\eta} \\ &\quad - \eta\left(\|\boldsymbol{\omega}\| - \frac{k\gamma + k\alpha\beta + d_{\max}}{2\eta}\right)^2 \\ &\leqslant -[k(1-\alpha)-\eta]\|\boldsymbol{\omega}\|^2 + \frac{(k\gamma + k\alpha\beta + d_{\max})^2}{4\eta} \end{aligned} \tag{18}$$

It follows from Property 1 in (4) that

$$\|\boldsymbol{\omega}\|^2 \geqslant \frac{1}{J_{\max}}\boldsymbol{\omega}^T\boldsymbol{J}\boldsymbol{\omega} \tag{19}$$

Substituting (19) into (18) leads

$$\begin{aligned} \dot{V} &\leqslant \frac{2[k(1-\alpha)-\eta]}{J_{\max}}\left[k\beta(1-q_0)^2 + k\beta\boldsymbol{q}_v^T\boldsymbol{q}_v\right] + \frac{(k\gamma + k\alpha\beta + d_{\max})^2}{4\eta} \\ &\quad - \frac{2[k(1-\alpha)-\eta]}{J_{\max}}\left[\frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{J}\boldsymbol{\omega} + k\beta(1-q_0)^2 + k\beta\boldsymbol{q}_v^T\boldsymbol{q}_v\right] \\ &= \frac{4k\beta[k(1-\alpha)-\eta]}{J_{\max}}(1-q_0) + \frac{(k\gamma + k\alpha\beta + d_{\max})^2}{4\eta} \\ &\quad - \frac{2[k(1-\alpha)-\eta]}{J_{\max}}\left[\frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{J}\boldsymbol{\omega} + k\beta(1-q_0)^2 + k\beta\boldsymbol{q}_v^T\boldsymbol{q}_v\right] \end{aligned} \tag{20}$$

Considering the candidate of Lyapunov function V in (10), the above inequality can be rewritten as

$$\dot{V} \leqslant -\frac{2[k(1-\alpha)-\eta]}{J_{\max}}V + \frac{8k\beta[k(1-\alpha)-\eta]}{J_{\max}} + \frac{(k\gamma + k\alpha\beta + d_{\max})^2}{4\eta} \tag{21}$$

In view of above inequality, it follows that

$$\dot{V} \leqslant -\varepsilon V + \varphi \tag{22}$$

where

$$\varepsilon = \frac{2[k(1-\alpha)-\eta]}{J_{\max}}, \quad \varphi = \frac{8k\beta[k(1-\alpha)-\eta]}{J_{\max}} + \frac{(k\gamma + k\alpha\beta + d_{\max})^2}{4\eta} \tag{23}$$

Therefore, it can be concluded that the event-triggered attitude stabilization control system is uniformly ultimately bounded with the ultimate bounds $\|\boldsymbol{\omega}\| \leqslant \sqrt{2\varphi/(J_{\min}\varepsilon)}$ and $\|\boldsymbol{q}_v\| \leqslant \sqrt{\varphi/(k\beta\varepsilon)}$.

The triggering sequence t_i for $i = 0, 1, 2, \ldots$ is admissible if the inter-update times are lower bounded by a positive value. The event-triggered mechanism should avoid the phenomenon of accumulation of consecutive triggering instants. This phenomenon is called Zeno behavior (Postoyan et al., 2015). This is critical for spacecraft attitude control as control torque cannot be executed continuously in practice. The admissibility guarantees that the generated triggering sequence is feasible in practice. It is proved in the following Theorem that the triggering sequence determined by (8) is admissible. □

Theorem 2. *Consider the spacecraft attitude stabilization system (1)–(3) with Assumption 1 and the control law (6). Then, the inter-update time T_i implicitly defined by the event-triggered rule (8) is lower bounded by a positive value.*

Proof. The $(i+1)^{th}$ inter-update times $T_i = t_{i+1} - t_i$ is the time it takes for $\|\boldsymbol{e}\|$ to increase from 0 to $\alpha\|\boldsymbol{s}\| + \gamma$. Define $\Gamma := \{t \in [t_i,\ t_{i+1}) : \|\boldsymbol{e}(t)\| = 0\}$. For $t \in [t_i,\ t_{i+1}) \setminus \Gamma$

$$\frac{d}{dt}\|\boldsymbol{e}(t)\| \leqslant \left\|\frac{d}{dt}[\boldsymbol{s}(t_i) - \boldsymbol{s}(t)]\right\| = \left\|\frac{d}{dt}\boldsymbol{s}(t)\right\| \tag{24}$$

By using (1), (2) and (5), it is obtained that

$$\boldsymbol{J}\dot{\boldsymbol{s}} = -\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} + \frac{1}{2}k\boldsymbol{J}(\boldsymbol{q}_v^{\times} + q_0\boldsymbol{I}_3)\boldsymbol{\omega} + \boldsymbol{u} + \boldsymbol{d} \tag{25}$$

Define

$$\boldsymbol{L}(\cdot) = -\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} + \frac{1}{2}k\boldsymbol{J}(\boldsymbol{q}_v^{\times} + q_0\boldsymbol{I}_3)\boldsymbol{\omega} + \boldsymbol{d} \tag{26}$$

As shown in Shen et al. (2015) and Cai et al. (2008), there exist a positive constant b such that

$$\|\boldsymbol{L}(\cdot)\| \leqslant b\Phi \tag{27}$$

$$\Phi = 1 + \|\boldsymbol{\omega}\| + \|\boldsymbol{\omega}\|^2 \tag{28}$$

Eq. (25) can be rewritten as

$$\boldsymbol{J}\dot{\boldsymbol{s}} = \boldsymbol{L}(\cdot) + \boldsymbol{u} \tag{29}$$

Substituting (29) into (24) yields

$$\frac{d}{dt}\|\boldsymbol{e}(t)\| \leqslant \left\|\boldsymbol{J}^{-1}\right\|\|\boldsymbol{L}(\cdot) + \boldsymbol{u}\| \tag{30}$$

Substituting the control law (6) into (30) leads to

$$\begin{aligned}\frac{d}{dt}\|\boldsymbol{e}(t)\| &\leqslant \left\|\boldsymbol{J}^{-1}\right\|\|\boldsymbol{L}(\cdot) - k\boldsymbol{s}(t_i)\| \\ &\leqslant \left\|\boldsymbol{J}^{-1}\right\|b\Phi + \left\|\boldsymbol{J}^{-1}\right\|\|-k\boldsymbol{s}(t_i)\| \\ &= \left\|\boldsymbol{J}^{-1}\right\|b\Phi + \left\|\boldsymbol{J}^{-1}\right\|\|-k\boldsymbol{s} - k\boldsymbol{e}\| \\ &\leqslant \left\|\boldsymbol{J}^{-1}\right\|b\Phi + k\left\|\boldsymbol{J}^{-1}\right\|\|\boldsymbol{s}\| + k\left\|\boldsymbol{J}^{-1}\right\|\|\boldsymbol{e}\|\end{aligned} \tag{31}$$

According to comparison Lemma in Khalil (2002), the solution to (31) with initial condition $\|\boldsymbol{e}(t)\| = 0$ can be derived as

$$\|\boldsymbol{e}(t)\| \leqslant \frac{b\Phi + k\|\boldsymbol{s}\|}{k}\left[\exp\left(k\left\|\boldsymbol{J}^{-1}\right\|(t - t_i)\right) - 1\right] \tag{32}$$

The time instant t_{i+1} is triggered once the triggering condition (8) is satisfied. Hence, it follows from (32) that

$$\alpha\|\boldsymbol{s}(t_{i+1})\| + \gamma = \|\boldsymbol{e}(t_{i+1})\| \leqslant \frac{b\Phi + k\|\boldsymbol{s}(t_{i+1})\|}{k}\left[\exp\left(k\left\|\boldsymbol{J}^{-1}\right\|T_i\right) - 1\right] \tag{33}$$

Hence, the inter-update times are uniformly lower bounded by T_i

$$T_i \geqslant \frac{1}{k\left\|\boldsymbol{J}^{-1}\right\|}\ln\left(1 + \frac{k\|\boldsymbol{e}(t_{i+1})\|}{b\Phi + k\|\boldsymbol{s}(t_{i+1})\|}\right) \tag{34}$$

From the triggering condition in (8), it follows that

$$\|\boldsymbol{e}(t_{i+1})\| \geqslant \gamma \tag{35}$$

Hence, (34) can be written as

$$T_i \geqslant \frac{1}{k\left\|\boldsymbol{J}^{-1}\right\|}\ln\left(1 + \frac{k\gamma}{b\Phi + k\|\boldsymbol{s}(t_{i+1})\|}\right) \tag{36}$$

Since $\boldsymbol{\omega}$ converges to a small set containing the origin as shown in Theorem 1, $b\Phi$ is upper bounded by positive constant. Furthermore, $k,\ \delta, \gamma$, and $\left\|\boldsymbol{J}^{-1}\right\|$ are all positive. Thus, it can be concluded that the inter-update times T_i are always lower bounded away from zero. □

Remark 5. Theorem 2 guarantees a minimum positive lower bound for the inter-update times T_i. This is essential for the event-triggered control scheme to get applied to spacecraft attitude control system. Hence, the event-triggered rule (8) guarantees the stability of closed-loop system with respect to the measurement errors $\boldsymbol{e}$ in the presence of external disturbances. Furthermore, the absence of Zeno phenomenon of triggering instants is guaranteed.

5. Illustrative example

In this section, an example is given to validate the proposed event-triggered attitude stabilization control scheme for spacecraft. In this example, the inertia matrix of spacecraft $\boldsymbol{J}$ is assumed to be as follows:

$$\boldsymbol{J} = \begin{bmatrix} 147 & 6.5 & 6 \\ 6.5 & 158 & 5.5 \\ 6 & 5.5 & 137 \end{bmatrix} \text{kg} \times \text{m}^2, \tag{37}$$

The external disturbances used in the simulation are supposed to be the following form.

$$\boldsymbol{d}(t) = \begin{bmatrix} 1 + 2\sin(0.005t) \\ -1 - 5\cos(0.005t) \\ 2 - 4\cos(0.005t) \end{bmatrix} \times 10^{-4}\ \text{Nm} \tag{38}$$

The attitude angle measurement accuracy is assumed to be $0.002\ \text{deg}(1\sigma)$, and the angular velocity measurement accuracy is assumed to be $0.0002\ \text{deg/sec}(1\sigma)$. Both measurement errors are assumed to be normally distributed.

In accordance with the method in Wie et al. (1989), the control gains in (6) are designed as:

$$\begin{aligned} &k = 2\xi\omega_n\|\boldsymbol{J}\| \\ &k\beta/2 = \omega_n^2\|\boldsymbol{J}\| \\ &\omega_n = 8/t_s \end{aligned} \tag{39}$$

where the damping ratio ξ is chosen as $\xi = 1$, and the settling time t_s is chosen as $t_s = 100$ sec.

The parameters α and γ in (8) is chosen as $\alpha = 0.5$ and $\gamma = 1 \times 10^{-6}$, respectively. The maximum control torque is assumed to be 0.2 Nm. This constraint on control torque is considered in the simulation.

The attitude control errors and angular velocity errors by using the proposed controller are shown in Figs. 2 and 3 respectively. For a clear interpretation of the results, attitude control errors are expressed in Euler angles converted from unit quaternion in the simulation. It is observed that the attitude errors and angular velocity errors converge to a small bound. The control torque at transient and steady states are shown in Figs. 4 and 5, respectively. Evolution of $\|\boldsymbol{e}(t)\|$ and $\alpha\|\boldsymbol{s}\| + \gamma$ at transient and steady states are shown is plotted in Figs. 6 and 7, respectively. Obviously, the simulation results verify the theoretical analysis and demonstrate the effectiveness of the proposed event-triggered control scheme.

The storage size of each component of the filtered error $\boldsymbol{s}$ and control torque $\boldsymbol{u}$ in practice is usually chosen as 4 bytes. Simulation time in this example is chosen as 1000 s. Number of triggering instants with the proposed event-triggered control law is 84. Thus, data to be sent over the communication channel with the event-triggered strategy

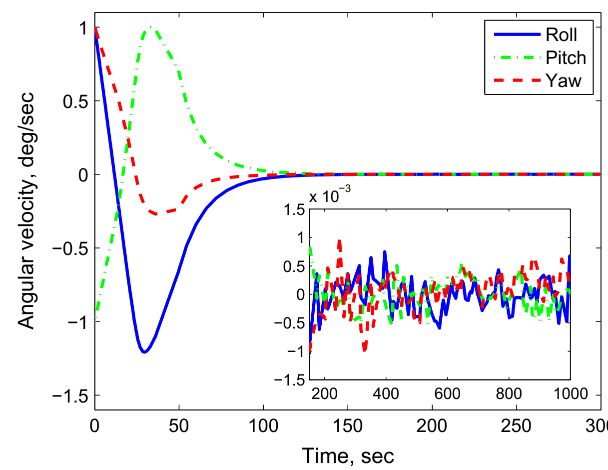

Fig. 3. Angular velocity error under the event-triggered control scheme.

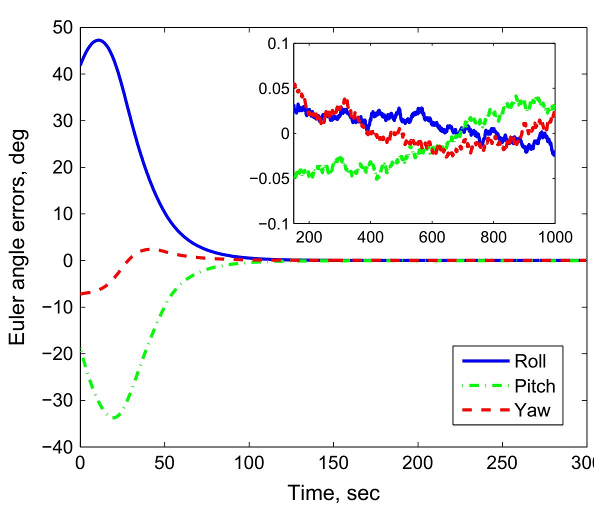

Fig. 2. Attitude error of spacecraft under the event-triggered control scheme.

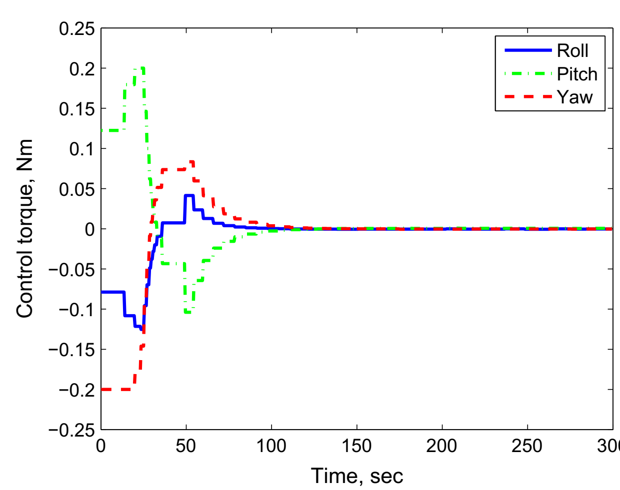

Fig. 4. Control torque $\boldsymbol{u}(t)$ under the event-triggered control scheme.

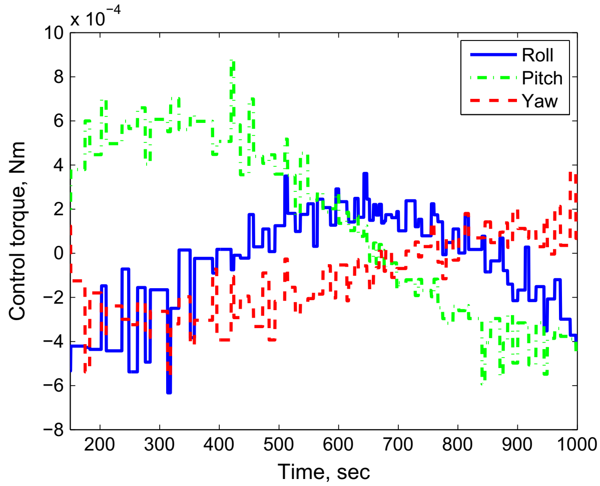

Fig. 5. Control torque $\boldsymbol{u}(t)$ at steady-state under the event-triggered control scheme.

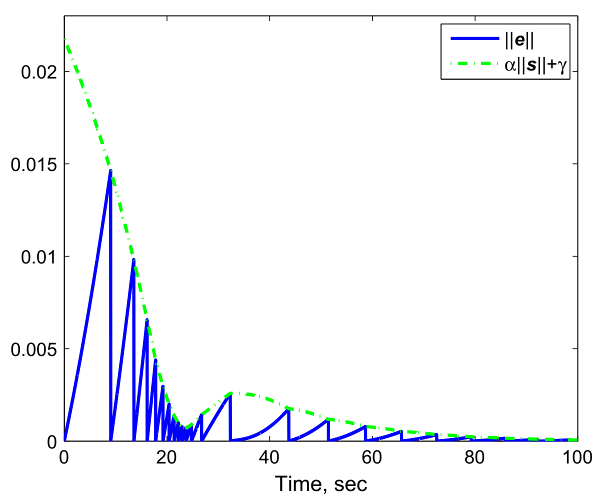

Fig. 6. Evolution of $\|e(t)\|$ and $\alpha\|s\| + \gamma$ under the event-triggered control scheme.

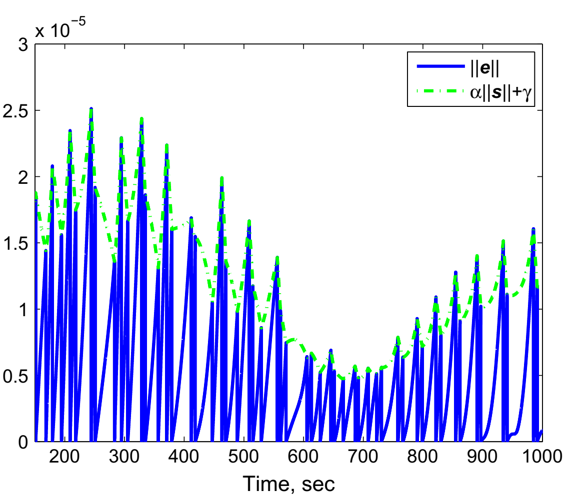

Fig. 7. Evolution of $\|e(t)\|$ and $\alpha\|s\| + \gamma$ at steady-state.

is 2016 bytes. In the conventional time-triggered attitude control system with the equivalent control performances of the case with event-triggered strategy, the sampling time is chosen as 1sec. Then, data to be sent over the communication channel without event-triggered strategy is 24,000 bytes. Thus, the size of communication data is greatly reduced by 91.6%.

6. Conclusions

An event-triggered control scheme is proposed in this paper for spacecraft attitude control system with limited communication and external disturbances. In the presented strategy, a trigger mechanism is employed to determine the event-triggering instants by evaluating the event-triggering condition in an approximately continuous way. Once the event-triggering condition is violated, the information of attitude $\boldsymbol{s}$ is transmitted to the attitude control module first and then the control torque $\boldsymbol{u}$ is updated and transmitted to the actuators of spacecraft. The ZOHs are employed to hold the last information until the next transmission is received. Simulation results show the effectiveness of the proposed event-triggered attitude stabilization control scheme. Data to be sent over the communication channel with the proposed event-triggered control law is greatly reduced.

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