Fuzzy Supervised Learning-Based Model-Free Adaptive Fault-Tolerant Spacecraft Attitude Control with Deferred Asymmetric Constraints

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***Abstract*—Aiming at solving the model-free fault-tolerant spacecraft attitude control problem, a data-driven adaptive control scheme is proposed to the spacecraft in the presence of actuator faults and performance constraints. First, the discrete-time attitude dynamic model is transformed into an affine nonlinear system based on local dynamic linearization. Then, a fuzzy logic-based lazily adapted constant kinky inference rule is introduced to predict the arbitrarily continuous nonlinear actuator faults and model uncertainties by supervised learning with insufficient prior knowledge. To satisfy time-varying deferred asymmetric constraints of the attitude tracking error, a virtual control law is proposed in the attitude control loop using the back-stepping approach, which is derived from a deferred switching transformation and barrier Lyapunov function. The stability of the data-driven adaptive fault-tolerant attitude control for the nonlinear discrete-time spacecraft attitude dynamics is analyzed rigorously with the aid of contraction mapping principle and discrete-time Lyapunov theory. Compared with existing methods, the proposed one considers nominal non-global Lipschitz nonlinear system with arbitrarily continuous unmodeled uncertainties and time-varying actuator faults, and achieves smaller prediction error bound than general kinky inference scheme and better closed-loop performance by estimating and compensating unknown dynamics. Finally, numerical simulation verifies the effectiveness of the proposed control scheme.**

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***Index Terms*—Data-driven control, spacecraft attitude control, adaptive control, barrier Lyapunov function, supervised learning, kinky inference, fault-tolerant control, contraction mapping principle.**

I. INTRODUCTION

HIGH precision spacecraft attitude control is a problem of great challenge in many practical space missions [1]–[3], such as in-orbit servicing and deep space exploration. However, due to multiple inertia uncertainties, complex external disturbances and actuator faults, severe uncertainties and nonlinearity may exist, leading to difficulties in obtaining an accurate spacecraft model. Therefore, model-based control scheme may not be suitable for high precision spacecraft attitude control problem, though many works has been developed in [4]–[7].

To reject the nonlinearities, disturbances and actuator faults, a few data-driven results leveraging adaptive control technique have been reported in [8]–[14]. In [8], a data-driven adaptive control scheme is proposed for state constrained nonlinear systems when considering hardware requirements. In [11], a data-driven active disturbance rejection control (ADRC) scheme is proposed, which is effective in dealing with the nonlinearities by embedding an adaptive estimation in the nonaffine nonlinear system under the globally Lipschitz assumption. Recently, a data-driven immersion and invariance adaptive control scheme is proposed to the attitude control of a rigid body spacecraft, which tackles the attitude and angular rate constraints [12]. However, when disturbances and actuator faults are taken into consideration in spacecraft attitude control systems, the data-driven adaptive approach is rarely developed.

For the problem of the spacecraft attitude control, many aforementioned works have been reported considering the eminent control approach of fault-tolerant control (FTC), aiming at ensuring the state or output performance when faults occur on the spacecraft. Strategies of FTC scheme are generally divided into active FTC and passive FTC [15]. For passive FTC, the robust control scheme is often considered, both for healthy and fault cases. For active FTC, the fault detection and diagnosis (FDD) is introduced to the control process, and the controller is retrofitted by the fault reconfiguration mechanism [5]. Recent works on the determination of the fault reconfiguration mechanism are summarized as adaptive control approach [6], control allocation [3], fault observation [2] and prescribed performances control (PPC) [16]. Recently, the data-driven learning-based fault observation is considered in the FTC design [13]. However, considering insufficient prior datasets of output measurements in practical spacecraft attitude systems, the enhanced learning-based fault observation and FTC system are required urgently.

Supervised learning method is a class of model estimation method with great flexibility and ability to approximate functions [17], [18]. For nonaffine globally Lipschitz systems, Lipschitz interpolation (LI) [19]–[23]

and nonlinear set membership (NSM) [24] methods are efficient to estimate the model uncertainties by setting a rational value of the Lipschitz constant to the predictor, which is acted as a hyper-parameter. However, for the model-free case, a novel design of LI containing constant estimation, which is referred to lazily adapted constant kinky inference (LACKI), is needed when insufficient prior knowledge is provided and the bounded estimation error is required [23], [25]. Although LACKI method has not yet been validated in spacecraft attitude control systems, it has been applied in the model predictive control [25] and model reference adaptive control [23], where the prediction performance is well verified in the simulated aircraft roll dynamics.

However, the aforementioned LACKI learning rule is proposed based on the assumption that the target function to be predicted is Lipschitz continuous [23]. Although the prediction boundedness is still satisfied in the non-Lipschitz continuous case, the LACKI predictor will lose effectiveness with wider prediction error bound [25]. Fuzzy logic systems (FLSs) [26] are efficient approach to deal with the non-Lipschitz continuous uncertainties with faster convergence. Depending on the IF-THEN rules of the FLSs, the uncertainties can be completely approximated, usually combining with the adaptive laws to obtain the estimation of the fuzzy weight [27], [28]. Many applications of FLSs have been investigated for the constrained control problem [26], stochastic nonlinear systems [27], [28] and data-driven approach [29], to overcome the impact of disturbances or uncertainties in practical systems. To enhance the effectiveness of the LACKI predictor and extend the prediction ability to the non-Lipschitz continuous condition, it is worth mentioned that the introduction of FLSs should be a feasible scheme, which has not yet been reported in the related research.

Constrained control problem of the spacecraft attitude system has attracted a great deal of interest recently [16], [30]–[33]. To guarantee the performance constraints during the control progress, several control approaches are proposed, such as prescribed performance control and barrier Lyapunov function. Prescribed performance control approach presets the overshoot of the tracking error, the steady and transient performance and it has been widely applied to spacecraft attitude control [16], [30], [33]. In [16], a fault-tolerant control scheme is proposed, by combining the prescribed performance control and adaptive approach considering the input saturation. In [33], a PD-like model-free prescribed performance control scheme is introduced to the attitude tracking control of flexible spacecraft subject to predefined tracking error constraints. Barrier Lyapunov function-based methods are verified to be efficient on constrained control for a class of strict-feedback nonlinear systems, which are mainly subjected to constant or time-varying state constraints [31], [32]. Taking advantages of the barrier Lyapunov function, a constrained controller consists of a nominal state feedback and a compensator is derived to adjust the state response into the performance bound. In [31], a tangent barrier Lyapunov–Krasovskii function combined with a neural network unit is utilized for the strict feedback systems to deal with nonlinear uncertainties and undetectable actuation faults despite the presence of output constraint and prescribed transient performance constraint. In [32], an asymmetric barrier Lyapunov function with relaxed state constraint at the initial phase is developed by utilizing error-shifting transformation.

To achieve precise attitude tracking of spacecraft subject to unmodeled dynamics, actuator faults and deferred asymmetric performance constraints, a fuzzy supervised learning-based data-driven adaptive back-stepping control scheme is proposed, where a fuzzy lazily adapted constant kinky inference (FLACKI) rule-based predictor is developed to learn the external disturbances and actuator faults and a discrete time adaptive law is utilized to estimate the input gain, respectively. The main contributions of this paper are summarized as follows:

1) Incorporated with the fuzzy logic, the proposed novel supervised FLACKI learning rule enhances the online-learning capability of the existing LACKI method in [23]. More importantly, it is applicable to a more general class of arbitrary continuous systems, and the upper bound of the prediction error can be reduced when there exists the over-fitting phenomenon.
2) A data-driven control scheme using barrier Lyapunov function is developed in the outer loop of the back-stepping-based control framework to deal with the deferred asymmetric performance constraint, which is a more general case that the initial tracking condition is unknown and the upper and lower performance bounds are set to be different, resulting in more feasibility to be implemented in practical engineering than the existing constrained control scheme [33].

The rest of the paper is organized as follows. In Section II, attitude dynamic of the rigid spacecraft with actuator faults is provided, and is further discretized by using local dynamic linearization. In addition, the general LACKI rule is also introduced. A fuzzy logic-based FLACKI rule is formulated in Section III, and a supervised learning predictor is designed to estimate model uncertainties. In Section IV, the data-driven constrained adaptive fault-tolerant attitude controller is designed by leveraging the barrier Lyapunov function and back-stepping technique. In Section V, the effectiveness of the proposed control scheme is demonstrated via numerical simulations. Section VI draws the conclusions.

II. PROBLEM FORMULATION

In this section, modeling of spacecraft motion with actuator faults and external disturbances are derived, then the local dynamic linearization is introduced to obtain a nonaffine discrete-time model. Finally, the existing LACKI rule is introduced.

A. Modeling of Spacecraft Attitude Dynamics

The attitude kinematics and dynamics are expressed as follows:

$$\dot{q}_0 = -\frac{1}{2} q^{\mathrm{T}} \omega, \tag{1a}$$

$$\dot{q} = \frac{1}{2} \left(q^{\times} + q_0 I_3 \right) \omega, \tag{1b}$$

$$J\dot{\omega} = -\omega^{\times} J \omega + \tau + d, \tag{1c}$$

where $J = J^{\mathrm{T}} \in \mathbb{R}_{3\times3}$ denotes the positive-definite inertia matrix of the rigid spacecraft, $\omega \in \mathbb{R}_3$ is the angular velocity of the spacecraft relative to the inertial coordinate system expressed in the body coordinate system, $Q = \left[q^{\mathrm{T}}, q_0\right]^{\mathrm{T}} \in \mathbb{R}_4$ is the unit quaternion describing the pointing direction in the body coordinate system relative to the inertial coordinate system, $\tau = [\tau_x, \tau_y, \tau_z]^{\mathrm{T}} \in \mathbb{R}_3$ and $d \in \mathbb{R}_3$ represent the control torque and external disturbance, respectively. The symbol $q^{\times} \in \mathbb{R}_{3\times3}$ is used to express a skew-symmetric matrix associated with the vector $q = [q_1, q_2, q_3]^{\mathrm{T}}$:

$$q^{\times} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}. \tag{2}$$

Assumption 2.1: The spacecraft inertia matrix J is uncertain and expressed as:

$$J = J_0 + \Delta J, \tag{3}$$

where $J_0 \in \mathbb{R}_{3\times3}$ is the unknown positive constant inertia part, and $\Delta J \in \mathbb{R}_{3\times3}$ denotes the unknown time-varying part containing unmodeled uncertainties. Both J_0 and ΔJ are assumed to be bounded.

B. Modeling of Actuator Faults

Taking into consideration two types of actuator faults in attitude controller design, i.e., gain fault and bias fault, the actual input $u = [u_1, u_2, u_3]^{\mathrm{T}} \in \mathbb{R}_3$ generated by 3 actuators is given by

$$u = Eu_c + \bar{u}, \tag{4}$$

where $u_c = [u_{c,1}, u_{c,2}, u_{c,3}]^{\mathrm{T}}$ is the nominal input without fault, $E = \mathrm{diag}\left([e_1, e_2, e_3]\right)$ is the unknown time-varying efficiency loss coefficient with $0 \le e_i < 1$ and $i = 1, 2, 3$, $\bar{u} = [\bar{u}_1, \bar{u}_2, \bar{u}_3]^{\mathrm{T}} \in \mathbb{R}_3$ is the unknown time-varying bias fault.

Then, the control torque acted on the spacecraft (cf. τ in (1c)) can be obtained as $\tau = u_c$. Therefore, the spacecraft attitude dynamics with actuator faults is expressed as:

$$J\dot{\omega} = -\omega^{\times} J \omega + Eu_c + \bar{u} + d. \tag{5}$$

C. Nonlinear Discrete-Time System Model and Local Dynamic Linearization

In practical, an approximated spacecraft attitude dynamics can only obtained according to I/O measurements instead of accurate model due to modeling uncertainties, disturbances, and faults [33]. Therefore, the attitude kinematics and faulty dynamics in (1a), (1b) and (5) is converted to a data-based form defined as the following nonaffine discrete time expression:

$$q_0(k+1) = q_0(k) - \frac{T}{2} q^{\mathrm{T}}(k)\,\omega(k), \tag{6a}$$

$$q(k+1) = q(k) + \Omega(k)\,\omega(k), \tag{6b}$$

$$\begin{aligned}\omega(k+1) = {} & f(\omega(k), ..., \omega(k-n_\omega), u_c(k), ..., u_c(k-n_u)) \\ & + h\left(d(k-1), ..., d(k-n_d)\right), \end{aligned} \tag{6c}$$

where T is the sampling step size, $\Omega(k) = \frac{T}{2} q^{\times}(k) + q_0(k) I_3 \in \mathbb{R}_{3\times3}$, $f(\omega(k), ..., \omega(k-n_\omega), u_c(k), ..., u_c(k-n_u)) = \omega(k) - J^{-1}(k)(\omega^{\times}(k)J(k)\omega(k) - E(k)u_c(k))$ is the vector containing the system states and inputs, $h(d(k-1), ..., d(k-n_d)) = \bar{u}(k) + d(k)$ denotes the nonlinear system disturbance, $n_\omega, n_u, n_d \in \mathbb{N}$ represent the linearization length. Considering the uncertain and unknown terms $J(k)$, $E(k)$, $\bar{u}(k)$ and $d(k)$ in above attitude control system, the two functions $f(\cdot), h(\cdot)$ are nonlinear and cannot be obtained analytically. To begin the controller design, the following assumptions are given:

Assumption 2.2: The actual input rate and control torque generated by n actuators have the following constraints:

$$|u_{c_i}(k) - u_{c_i}(k-1)| \le \rho_u, \;\; |u_{c_i}(k)| \le \rho_{u_c} \tag{7}$$

where $\rho_u > 0, \rho_{u_c} > 0$ are the upper bound of the i-th control torque and input rate for all $i \in \{1, \ldots, n\}$.

Assumption 2.3: In the discrete-time nonlinear system (6c), the function $f(\cdot)$ satisfies the global Lipschitz condition with respect to the state $\omega(k)$. That is, for any two sampling points k_1 and k_2, there exists

$$\begin{aligned} & \| f(\omega_1(k_1), ..., \omega_P(k_1)) - f(\omega_1(k_2), ..., \omega_P(k_2)) \| \\ & \le L_1 \|\omega_1(k_1) - \omega_1(k_2)\| + ... + L_P \|\omega_P(k_1) - \omega_P(k_2)\|, \end{aligned} \tag{8}$$

where L_m is the assumed Lipschitz constant with $m \in \{1, ..., P\}$ and $P = n_\omega + n_d + n_u + 3$. The partial derivative $\partial f(\cdot)/\partial u_c(\cdot)$ exists with an unchanged sign, which expresses as $\partial f(\cdot)/\partial u_c(\cdot) \le -\epsilon_k$ or $\partial f(\cdot)/\partial u_c(\cdot) \ge \epsilon_k$ with ϵ_k being a small positive constant [9], [11].

Assumption 2.4: The function $h(\cdot)$ representing the disturbance is bounded and non-Lipschitz continuous. In practical, $h(\cdot)$ consists of the state-related term $h_1(\cdot)$ and bounded external term $h_2(d_0)$, i.e.,

$$\begin{aligned} & h\left(d(k-1), ..., d(k-n_d)\right) \\ & = h_1\left(\omega(k), ..., \omega(k-n_\omega)\right) \\ & + h_2\left(d_0(k-1), ..., d_0(k-n_{d_0})\right) \end{aligned}$$

where $\|h_2(d_0(k-1), ..., d_0(k-n_{d_0}))\| \le \bar{E}_h$.

Remark 2.1: **Assumption 2.1** and **Assumption 2.2** are common assumptions for a general spacecraft, due to the time-varying mass property of the spacecraft and the saturations of the input torque. **Assumption 2.3** and **Assumption 2.4** mainly refer to the local dynamic linearization of the discrete-time attitude dynamics considering the Lipschitz continuous $f(\cdot)$ and the non-Lipschitz $h(\cdot)$.

Especially, in **Assumption 2.4**, the non-Lipschitz $h(\cdot)$ is due to the actuator faults and external disturbances from the environment.

Then, according to the above assumptions and the local dynamic linearization in [9], [11], the nonlinear discrete-time model (6c) can be rewritten as:

$$\begin{aligned}&\Delta\omega\left(k+1\right)=f\left(\chi\left(k\right),u_c\left(k\right)\right)-f\left(\chi\left(k-1\right),u_c\left(k-1\right)\right)\\&-f\left(\chi\left(k\right),u_c\left(k-1\right)\right)+f\left(\chi\left(k\right),u_c\left(k-1\right)\right)\\&+h\left(\delta\left(k\right)\right)-h\left(\delta\left(k-1\right)\right)\\&=\frac{\partial f\left(\cdot\right)}{\partial u_c\left(k\right)}\Delta u_c\left(k\right)+\zeta\left(k\right),\end{aligned}\tag{9}$$

where

$$\begin{aligned}&\chi\left(k\right)=\left[\omega\left(k\right),...,\omega\left(k-n_\omega\right),u_c\left(k-1\right),...,u_c\left(k-n_u\right)\right],\\&\delta\left(k\right)=\left[d\left(k-1\right),...,d\left(k-n_d\right)\right],\\&\zeta\left(k\right)=-f\left(\chi\left(k-1\right),u_c\left(k-1\right)\right)+f\left(\chi\left(k\right),u_c\left(k-1\right)\right)\\&+h\left(\delta\left(k\right)\right)-h\left(\delta\left(k-1\right)\right),\end{aligned}$$

and Δ is the difference operator defined as

$$\Delta x\left(k+1\right)=x\left(k+1\right)-x\left(k\right).\tag{10}$$

Denote $\Theta^{\mathrm{T}}\left(k\right)=\frac{\partial f(\cdot)}{\partial u_c(k)}\in\mathbb{R}_{3\times3}$, according to the **Assumption 2.3**, $\Theta^{\mathrm{T}}\left(k\right)$ is bounded with the upper bound of $\rho_\Theta>0$.

Then, the discrete dynamics in (6c) is transformed into the following local dynamic linearization form:

$$\Delta\omega\left(k+1\right)=\Theta^{\mathrm{T}}\left(k\right)\Delta u_c\left(k\right)+\zeta\left(k\right).\tag{11}$$

D. LACKI Rule Formulation

Before giving the control framework, the LACKI rule is introduced and formulated first. The LACKI rule is a supervised learning rule that can achieve bounded model prediction [23]. Define a pseudo-metric $\partial:\mathcal{X}^2\rightarrow\mathbb{R}_{\geq0}$ and a output pseudo-metric $\partial_{\mathcal{Y}}:\mathcal{Y}^2\rightarrow\mathbb{R}_{\geq0}$ which are respectively assigned to the d-dimensional normed input space $\mathcal{X}$ and the m-dimensional normed output space $\mathcal{Y}$.

The Lipschitz continuous function set $Lip(L)$, is defined as in [23] with the introduction of the Lipschitz constant L. We also denote $f\in Lip\left(L^*\right)$ as the continuous function with the smallest Lipschitz constant L^*, and choose $f:\mathcal{X}\rightarrow\mathcal{Y}$ as the target function of the supervised learning rule [20]. During the initial steps, the available prior sample dataset $\mathcal{B}_n:=\left\{\left(s_i,\tilde{f}_i\right)|i=1,...,N_n\right\}$ is pre-given, which contains $N_n\in\mathbb{N}$ sample input-output pairs $\left(s_i,\tilde{f}_i\right)$, where the target function $\tilde{f}_i\in\mathcal{Y}$ at sample inputs $s_i\in\mathcal{X}$.

At the n-th step, we define $\mathcal{K}_n=\{s_i|i=1,...,N_n\}\subset\mathcal{X}$ as the grid of s_i and $\mathcal{Y}_n=\left\{\tilde{f}_i|i=1,...,N_n\right\}\subset\mathcal{Y}$ as the output sequence, then the dataset $\mathcal{B}_n$ is further expressed as $\mathcal{B}_n=\left(\mathcal{K}_n,\mathcal{Y}_n\right)$. To carry out learning rule or inference procedure for constructing the predictor, the definition of LACKI rule-based on nonparametric regression is given firstly.

Definition 2.1 (LACKI rule) [23]: For a prior set of samples $\mathcal{B}_n$ at the n-th step, assign $\tilde{\partial}\left(\cdot,\cdot;\Xi\left(n\right)\right):\mathcal{X}^2\rightarrow\mathbb{R}$ parameterized by the hyper-parameter $\Xi\left(n\right)$. Then the LACKI predictor is defined as:

$$\hat{f}_{n,r}\left(x;\Xi\left(n\right),\mathcal{B}_n\right):=\frac{1}{2}m_{n,r}\left(x;\Xi\left(n\right)\right)+\frac{1}{2}w_{n,r}\left(x;\Xi\left(n\right)\right)\tag{12}$$

where

$$\begin{aligned}&m_{n,r}\left(\cdot;\Xi\left(n\right)\right),w_{n,r}\left(\cdot;\Xi\left(n\right)\right):\mathcal{X}\rightarrow\mathbb{R}_m,\\&m_{n,r}\left(\cdot;\Xi\left(n\right)\right):=\min\nolimits_{i=1,...,N_n}\tilde{f}_{i,r}+\tilde{\partial}\left(x,s_i;\Xi\left(n\right)\right),\\&w_{n,r}\left(\cdot;\Xi\left(n\right)\right):=\max\nolimits_{i=1,...,N_n}\tilde{f}_{i,r}-\tilde{\partial}\left(x,s_i;\Xi\left(n\right)\right),\end{aligned}$$

$\hat{f}_{n,r}$ and $\tilde{f}_{i,r}$ denote the r-th component of the output $\hat{f}_n$ and $\tilde{f}_i$, $\tilde{\partial}\left(x,x';L\left(n\right)\right)=L\left(n\right)\partial\left(x,x'\right)$ with $L\left(n\right)$ being the Lipschitz constant to be estimated. To simplify the exposition of the learning rule establishment, suppose that the output dimension norm space $\mathcal{Y}\subseteq\mathbb{R}_m$, of which the pseudo metric satisfies $\partial_{\mathcal{Y}}\left(y,y'\right)=\left\|y-y'\right\|_\infty,\forall y,y'\in\mathcal{Y}$, and the input dimension norm space $\mathcal{X}\subseteq\mathbb{R}_d$, of which the pseudo metric satisfies $\partial\left(x,x'\right)=\left\|x-x'\right\|_\infty,\forall x,x'\in\mathcal{X}$.

The adapted update rule of the Lipschitz constant $L\left(n\right)$ is given as

$$L\left(n\right):=\max\left\{0,\max_{(s,s')\in U_n}\frac{\left\|\tilde{f}\left(s\right)-\tilde{f}\left(s'\right)\right\|_\infty-\lambda}{\left\|s-s'\right\|_\infty}\right\},\tag{13}$$

where $\lambda\geq0$ is a hyper-parameter, which is typically utilized to compensate for predictor drift due to the observation error and avoid $L\left(n\right)$ diverging or over-fitting [25]. Meanwhile, for the input set $S,S'\subset\mathcal{X}$, define $U\left(S,S'\right):=\{(s,s')\in S\times S'|\left\|s-s'\right\|_\infty>0\}$, where $U_n:=U\left(\mathcal{K}_n,\mathcal{K}_n\right)$ is the set of all distinct sample input pairs. To realize the online learning of Lipschitz constant $L\left(n\right)$, incremental learning is proposed to construct iteration from $L\left(n\right)$ to $L\left(n+1\right)$, so that the number of samples under the $(n+1)$-th step satisfies $\mathcal{K}_{n+1}=\mathcal{K}_n\cup\left\{s_{n+1}\right\},\forall n$. For $n\in\mathbb{N}$, choose $L\left(0\right):=0$, we inductively define the following incremental update rules:

$$\begin{aligned}&L\left(n+1\right)=\max\left\{L\left(n\right),\right.\\&\left.\max_{(s,s')\in U(\mathcal{K}_n,\{s_{n+1}\})}\frac{\left\|\tilde{f}\left(s\right)-\tilde{f}\left(s'\right)\right\|_\infty-\lambda}{\left\|s-s'\right\|_\infty}\right\}.\end{aligned}\tag{14}$$

To assess the LACKI rule, theoretical analysis of the boundedness of the predictor (14) has been established in [23]. Note that the error sequence is increasingly dense due to a query input set $\mathcal{I}\subseteq\mathcal{X}$. The definition of dense considering the grids sequence $\left(\mathcal{K}_n\right)_{n\in\mathbb{N}}$ is firstly given before the conclusion of the prediction boundedness.

Definition 2.2 [20]: For the grid sequence $\left(\mathcal{K}_n\right)_{n\in\mathbb{N}}$, if points in $\mathcal{K}_n$ can be used to approximate any point in $\mathcal{I}\subseteq\mathcal{X}$ with increasing accuracy, then the grid sequence is defined to become dense with respect to $\mathcal{I}$ with the limitation of sample points n. This can be formally written as

$$\text{If }\forall\epsilon_0>0,x\in\mathcal{I},\exists n_0\forall n\geq n_0,\exists z\in\mathcal{K}_n:\left\|x-z\right\|_\infty<\epsilon_0.$$

Moreover, if the coverage rate of the sequence to the predefined set is not related to x, the sequence is defined

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to become dense uniformly, with

Iff $\forall\epsilon_0 > 0, \exists n_0 \forall n \geq n_0, x \in \mathcal{I}, \exists z \in \mathcal{K}_n : \|x - z\|_\infty < \epsilon_0$.

According to **Definition 2.1**, the following theorem is given about the consistency of the LACKI prediction for the Lipschitz continuous target functions:

Theorem 2.1 (Uniform boundedness) [21]: Given a sample sequence $(\mathcal{B}_n)_{n\in\mathbb{N}}$, which exists an observation error with the upper bound of $\bar{e} \in \mathbb{R}_{\geq 0}$. For any $p > 0$, the hyper-parameter is set to be $\lambda := 2\bar{e} + p$. Defining a compact set $\mathcal{I} \subseteq \mathcal{X}$, the LACKI rule is considered to be a universal estimator if $(\mathcal{K}_n)_{n\in\mathbb{N}}$ converges uniformly to $\mathcal{I} \subseteq \mathcal{X}$, and the prediction sequence $\left(\hat{f}_n\right)_{n\in\mathbb{N}}$ is considered to be convergence to any continuous $f : \mathcal{X} \to \mathbb{R}_m$. When f satisfies Lipschitz continuous condition, the estimation error of LACKI converges uniformly to the upper bound of $2\bar{e} + \frac{p}{2}$. When f contains non-Lipschitz but continuous terms, the LACKI predictor can achieve uniform convergence bounded error with the upper bound of $2\bar{e} + \frac{3p}{2}$.

The detailed proof of **Theorem 2.1** can be found in [21], [23].

E. Control Objective

The control objective of this article is to propose a supervised learning-based strategy on the basis of the nominal iteration-learning-based control scheme, to deal with the non-Lipschitz disturbances, uncertainties and actuator faults of the spacecraft attitude dynamics, since the nominal iteration learning-based data-driven controller cannot handle the Lipschitz continuous uncertainties. Assuming that insufficient prior datasets can be obtained, a fuzzy logic based supervised learning predictor is proposed to achieve the fault observation, and a constrained control scheme is utilized based on the control barrier function to ensure the predfined control performance.

III. FUZZY SUPERVISED LEARNING-BASED PREDICTOR

In this section, a novel FLACKI rule is firstly derived by taking advantages of the fuzzy logic system, then a fuzzy supervised learning-based predictor is proposed to estimate the model uncertainties, leading to bounded prediction error.

A. FLACKI Rule Formulation

According to the formulation of LACKI rule, the boundedness of the LACKI prediction is guaranteed under the assumption of Lipschitz continuous target output, which is denoted as $f \in Lip(L^*)$ with the smallest Lipschitz constant L^*. However, when it comes to the condition of non-Lipschitz continuous, as given in [14], the general LACKI prediction accuracy seems to be reduced with the prediction error bound up to $2\bar{e} + \frac{3p}{2}$, where the chosen of p is decided by the hyper-parameter λ as $p = 2\bar{e} - \lambda$ to exclude the over-fitting phenomenon [25].

To extend the LACKI rule to an arbitrarily continuous target function with proper estimation error, an enhanced LACKI rule-based on fuzzy system is proposed in this section. Note that the fuzzy system is an efficient scheme to deal with the non-Lipschitz condition. Firstly, a fuzzy logic system is introduced to approximate the output $g \in \mathbb{R}_g$ with the following fuzzy IF-THEN inference rules [26], [34]:

$$\text{If} \quad x_1 \in \mathcal{F}_1^l, x_2 \in \mathcal{F}_2^l, ..., x_n \in \mathcal{F}_n^l, \quad \text{then} \quad g \in \mathcal{G}^l,$$

where $x = [x_1, x_2, ..., x_n]^{\mathrm{T}} \in \mathbb{R}_n$ expresses the system input, $l = 1, 2, ..., N$, N denotes the fuzzy rule number, $\mathcal{F}_i^l$ and $\mathcal{G}^l$ represent the fuzzy sets. According to [34], the fuzzy logic system can be obtained as

$$g(x) = \frac{\sum_{l=1}^{N} \vartheta_l \prod_{i=1}^{n} h_{\mathcal{F}_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} h_{\mathcal{F}_i^l}(x_i)\right]}, \tag{15}$$

where $\vartheta_l = \max_{g\in\mathbb{R}_g} h_{\mathcal{G}^l}(g)$, $\vartheta = [\vartheta_1, ..., \vartheta_N]$ denotes the fuzzy weight. $h_{\mathcal{G}^l}(g)$ and $h_{\mathcal{F}_i^l}(x)$ are the membership functions of $\mathcal{F}_i^l$ and $\mathcal{G}^l$, repectively. Let the fuzzy basis function be

$$\varphi_l(x) = \frac{\prod_{i=1}^{n} h_{\mathcal{F}_i^l}(x_i)}{\sum_{l=1}^{N} \left[\prod_{i=1}^{n} h_{\mathcal{F}_i^l}(x_i)\right]}. \tag{16}$$

Then $g(x)$ can be rewritten as

$$g(x) = \vartheta^{\mathrm{T}} \varphi(x), \tag{17}$$

where $\varphi(x) = [\varphi_1(x), \varphi_2(x), ..., \varphi_N(x)]$. Assuming that the input space $x_1, x_2, ..., x_n \in \mathcal{U}$ is a compact set in $\mathbb{R}_n$, for arbitrary $\epsilon^* > 0$, an optimal vector θ^* exists with

$$\sup_{x\in\mathcal{U}} \left\|g(x_n) - \theta^{*\mathrm{T}}\varphi(x_n)\right\|_\infty \overset{n\to\infty}{\to} \epsilon^*.$$

Similar to the discussion of the LACKI rule, define pseudo-metric $\partial : \mathcal{U}^2 \to \mathbb{R}_{\geq 0}$ and $\partial_{\mathcal{H}} : \mathcal{H}^2 \to \mathbb{R}_{\geq 0}$, then a set of arbitrarily continuous functions, which is denoted by the fuzzy logic system as $Fuz(\vartheta) = \{\phi : \mathcal{U} \to \mathcal{H} | \partial_{\mathcal{H}}(\phi(x), \phi(x')) \leq \|\vartheta\| \partial(\varphi(x), \varphi(x')), \forall x, x' \in \mathcal{U}\}$. Introduce the optimal vector θ^*, for a non-Lipschitz continuous target function g, there exists $g \in Fuz(\theta^*)$. With the pre-determined fuzzy basis function $\varphi(\cdot)$, the fuzzy weight should be finite.

When the target function g is unknown or consists of unmodeled dynamics, the supervised learning approach is proposed to the prediction of g with the assumption that an increasing amount of sample data set is obtained indexed by the sample step $n \in \mathbb{N}$ through the corresponding supervised learning rule, which is denoted as $\mathcal{D}_n := \{(s_i, \tilde{g}_i) | i = 1, ..., N_n\}$, where $\tilde{g}_i$ is the target output point at the sample input s_i. Similarly to the general Lipschitz continuous target function, we also assume that the samples output contains bounded observation error $\bar{e}$ and $\partial_{\mathcal{H}}(\tilde{g}_i, g(s_i)) \leq \partial_{\mathcal{H}}(0, \bar{e}(s_i))$.

With the above formulations of the target function, sample datasets, and the same definition of the sample input grid $\mathcal{G}_n = \{s_i | i = 1, .., N_n\} \subset \mathcal{U}$, a fuzzy predictor extending the general LACKI is proposed to obtain the evaluation of the prediction $\hat{g}_n(x)$ of $g(x)$.

Definition 3.1 (Fuzzy Lazily Adapted Constant Kinky Inference (FLACKI) rule): Consider an arbitrarily continuous target function $g : \mathcal{U} \to \mathcal{H}$, where $\mathcal{U}$ and $\mathcal{H}$ represent input space and the output space. For the input elements $x, x' \in \mathcal{U}$, with the assignment of the fuzzy basis function-based input space metric $\tilde{\partial}(\varphi(x), \varphi(x'); \Xi_1(n))$ parameterized by hyper-parameter $\Xi_1(n)$, the FLACKI predictor is formulated as

$$\begin{aligned}\hat{g}_{n,v}(x; \Xi(n), \mathcal{D}_n) := \frac{1}{2} u_{n,v}(\varphi(x), \varphi(s_i); \Xi_1(n)) \\ + \frac{1}{2} l_{n,v}(\varphi(x), \varphi(s_i); \Xi_1(n)),\end{aligned} \tag{18}$$

where

$$\begin{aligned}&u_{n,v}(\cdot; \Xi_1(n)), l_{n,v}(\cdot; \Xi(n)) : \mathcal{U} \to \mathbb{R}_m,\\ &u_{n,v}(\cdot; \Xi_1(n))\\ &:= \min_{i=1,\dots,N_n} \tilde{g}_{i,v} + \tilde{\partial}(\varphi(x), \varphi(s_i); \theta(n)),\\ &l_{n,v}(\cdot; \Xi_1(n))\\ &:= \max_{i=1,\dots,N_n} \tilde{g}_{i,v} - \tilde{\partial}(\varphi(x), \varphi(s_i); \theta(n)),\\ &\tilde{\partial}(\varphi(x), \varphi(s_i); \theta(n)) = \theta(n) \partial(\varphi(x), \varphi(s_i)),\\ &\partial(\varphi(x), \varphi(s_i)) = \|\varphi(x) - \varphi(s_i)\|_\infty, \forall x, s_i \in \mathcal{U},\end{aligned}$$

$\hat{g}_{n,v}$ and $\tilde{g}_{i,v}$ denote the v-th component of the output $\hat{g}_n$ and $\tilde{g}_i$, $\theta(n)$ represents the estimation of fuzzy weight, $\varphi(x)$ is derived as in (16). Similar to (13), for two sets $S, S' \subset \mathcal{U}$, define $\mathcal{C}(S, S') := \{(s, s') \in S \times S' | \|s - s'\|_\infty > 0\}$ with $\mathcal{C}_n := \mathcal{C}(\mathcal{G}_n, \mathcal{G}_n)$. The adapted iteration of the fuzzy weight $\theta(n)$ is given as

$$\begin{aligned}\theta(n+1) = \max\Big\{\theta(n), \\ \max_{(s,s') \in \mathcal{C}(\mathcal{G}_n, \{s_{n+1}\})} \frac{\|\tilde{g}(s) - \tilde{g}(s')\|_\infty - \lambda}{\|\varphi(s) - \varphi(s')\|_\infty}\Big\},\end{aligned} \tag{19}$$

where the initial value of the θ is set to be $\theta(0) = 0$.

To assess the FLACKI rule, theoretical analysis of the boundedness of the predictor (18) is established. For safety critical needs, the worst-case is in consideration. That is, for the prediction error sequence $\epsilon^\infty := (\epsilon_n^\infty)_{n \in \mathbb{N}}$, the worst-case with $\epsilon_n^\infty := \sup_{x \in \mathcal{U}} \|\hat{g}_n(x) - g(x)\|_\infty$ is studied. Note that the fuzzy basis function $\varphi(x)$ is strictly bounded by $\|\varphi(x)\| \le 1$. Then, according to **Definition 3.1**, with $\mathcal{G}_n$ becoming dense uniformly, the grid sequence of fuzzy basis function that can be denoted as $\varphi(x) \in \Phi_n$ is also seen as becoming dense or becoming dense uniformly. That is, points on Φ_n can be utilized to approximate any points in the value set of $\varphi(x)$ with increasing accuracy. The following theorem is given to show uniform boundedness of the FLACKI prediction for the arbitrarily continuous functions.

Theorem 3.1: With the pre-given sample sequence $(\mathcal{D}_n)_{n \in \mathbb{N}}$ and bounded observation error $\bar{e}$, selecting the hyper-parameter $\lambda := 2\bar{e} + p$ with $p > 0$ to avoid over-fitting, the FLACKI rule is considered to be a universal predictor as the fuzzy basis function grid Φ_n converges uniformly, and the predictor $(\hat{g}_n)_{n \in \mathbb{N}}$ can converge to any continuous target function with the prediction error upper bounded by $2\bar{e} + \frac{p}{2}$.

Proof: For any input $x \in \mathcal{U}$, let $\xi_n^x \in \arg\inf_{s \in \mathcal{G}_n} \|x - s\|_\infty$, which represents the nearest sample neighbor of x in the grid $\mathcal{G}_n$. For all $n \in \mathbb{N}, x \in \mathcal{U}$, the prediction error is calculated as

$$\begin{aligned}&\|\hat{g}_n(x) - g(x)\|_\infty\\ &\le \|\hat{g}_n(x) - g(\xi_n^x)\|_\infty + \|g(\xi_n^x) - g(x)\|_\infty\\ &\le \|\hat{g}_n(x) - \hat{g}_n(\xi_n^x)\|_\infty + \|g(\xi_n^x) - g(x)\|_\infty\\ &+ \|\hat{g}_n(\xi_n^x) - g(\xi_n^x)\|_\infty\\ &\le (\theta(n) + \vartheta^*) \|\varphi(x) - \varphi(\xi_n^x)\|_\infty + \|\hat{g}_n(\xi_n^x) - g(\xi_n^x)\|_\infty,\end{aligned} \tag{20}$$

where ϑ^* is denoted as $\vartheta^* = ||\theta^*||_\infty$. For $\|\hat{g}_n(\xi_n^x) - g(\xi_n^x)\|_\infty$, the sample-consistent of the LACKI rule need to be discussed. Let $j, k \in \{1, \dots, N_n\}$ and take $\tilde{g}_{i,v}$ as an example, for all $s_q \in \mathcal{G}_n, q \in \{1, .., N_n\}$, define

$$\begin{aligned}j \in \arg\min_i \tilde{g}_{i,v} + \theta(n) \|\varphi(s_i) - \varphi(s_q)\|_\infty,\\ k \in \arg\max_i \tilde{g}_{i,v} - \theta(n) \|\varphi(s_i) - \varphi(s_q)\|_\infty.\end{aligned}$$

According to (18), it can be obtained that

$$\begin{aligned}\hat{g}_{n,v}(s_q) = \frac{1}{2}\left(\tilde{g}_{j,v} + \theta(n) \|\varphi(s_j) - \varphi(s_q)\|_\infty\right) \\ + \frac{1}{2}\left(\tilde{g}_{k,v} - \theta(n) \|\varphi(s_k) - \varphi(s_q)\|_\infty\right).\end{aligned} \tag{21}$$

Since $\theta(n) \ge \max_{(s,s') \in \mathcal{C}_n} \frac{\|\tilde{g}(s) - \tilde{g}(s')\|_\infty - \lambda}{\|\varphi(s) - \varphi(s')\|_\infty}$, in particular, $\theta \ge \frac{|\tilde{g}_{k,v} - \tilde{g}_{q,v}| - \lambda}{\|\varphi(s_k) - \varphi(s_q)\|_\infty}$. Then, we further have

$$\theta \|\varphi(s_k) - \varphi(s_q)\|_\infty + \lambda \ge |\tilde{g}_{k,v} - \tilde{g}_{q,v}| = \tilde{g}_{k,v} - \tilde{g}_{q,v}. \tag{22}$$

Thus,

$$\begin{aligned}\tilde{g}_{j,v} + \theta(n) \|\varphi(s_j) - \varphi(s_q)\|_\infty \in [\tilde{g}_{q,v} - \lambda, \tilde{g}_{q,v}],\\ \tilde{g}_{k,v} - \theta(n) \|\varphi(s_k) - \varphi(s_q)\|_\infty \in [\tilde{g}_{q,v}, \tilde{g}_{q,v} + \lambda],\end{aligned} \tag{23}$$

holds, which further leads to

$$\hat{g}_{n,v}(s_q) \in \left[\tilde{g}_{q,v} - \frac{\lambda}{2}, \tilde{g}_{q,v} + \frac{\lambda}{2}\right].$$

Considering the existence of the observation error $e(s_q)$ with the upper bound $\bar{e}$, the prediction error on the sample point can be obtained as

$$\begin{aligned}&\|\hat{g}_n(s_q) - g(s_q)\|\\ &\le \|\hat{g}_n(s_q) - \tilde{g}(s_q)\| + \|\tilde{g}(s_q) - g(s_q)\|\\ &\le \frac{\lambda}{2} + \|e(s_q)\|_\infty \le \frac{\lambda}{2} + \bar{e}.\end{aligned} \tag{24}$$

Note that $\xi_n^x \in G_n$, (20) can be rewritten as

$$\begin{aligned}&\|\hat{g}_n(x) - g(x)\|_\infty\\ &\le (\theta(n) + \vartheta^*) \|\varphi(x) - \varphi(\xi_n^x)\|_\infty + \frac{\lambda}{2} + \bar{e}.\end{aligned} \tag{25}$$

From (25), the boundedness of LACKI prediction on $\mathcal{G}_n$ will be extended. Analysis is carried out respectively relative to the grid dense of the sample point. When the grid $\mathcal{G}_n$ becomes dense in the input domain $\mathcal{U}$, there is a rate function r_x that satisfies $\lim_{n \to \infty} r_x(n) = 0$ and

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$\|\varphi(x)-\varphi(\xi_n^x)\|_\infty \leq r_x(n)$. For all $n \in \mathbb{N}$, it can be obtained

$$\|\hat{g}_n(x) - g(x)\|_\infty \in \left[0, (\theta(n) + \vartheta^*) r_x(n) + \frac{\lambda}{2} + \bar{e}\right]. \quad (26)$$

Due to $\lim_{n\to\infty} r_x(n) = 0$, $\|\hat{g}_n(x) - g(x)\|_\infty$ converges to $\left[0, \frac{\lambda}{2} + \bar{e}\right]$.

On the other hand, in the case that the grid becomes dense uniformly, utilizing the uniform convergence with the independent rate $r(n)$, for some $\theta(n) = \bar{\theta} \in [0, \vartheta^*]$ and any $n \in \mathbb{N}$, it can be obtained that

$$\begin{aligned} &\sup_{x\in\mathcal{U}}\|\hat{g}_n(x) - g(x)\|_\infty \\ &\leq \left(\bar{\theta} + \vartheta^*\right) r(n) + \frac{\lambda}{2} + \bar{e} \overset{n\to\infty}{\to} \frac{\lambda}{2} + \bar{e}. \end{aligned} \quad (27)$$

As long as the hyper-parameter λ is set to be $\lambda := 2\bar{e} + p$ with $p \geq 0$, we have

$$\sup_{x\in\mathcal{X}}\|\hat{g}_n(x) - g(x)\|_\infty \overset{n\to\infty}{\to} \frac{p}{2} + 2\bar{e}.$$

This completes the proof. ■

B. FLACKI-Based Uncertainty Prediction

In this subsection, the uncertain term $\zeta(k)$ in local dynamic linearization model (11) is predicted by the FLACKI rule. Note that

$$\begin{aligned} \zeta(k) = &-f(\chi(k-1), u_c(k-1)) + f(\chi(k), u_c(k-1)) \\ &+ h(\delta(k)) - h(\delta(k-1)). \end{aligned} \quad (28)$$

Let the input vector be

$$\begin{aligned} s(k) = \Big[&\omega^{\mathrm{T}}(k), ..., \omega^{\mathrm{T}}(k - n_\omega - 1), \\ &u_c^{\mathrm{T}}(k-1), ..., u_c^{\mathrm{T}}(k - n_u - 1)\Big]^{\mathrm{T}}, \end{aligned} \quad (29)$$

according to **Assumption 2.3** and **Assumption 2.4**, $\zeta(k)$ is non-Lipschitz continuous but with an upper bound $\bar{\zeta}_l$ and contains a bounded partial term which is not related to the sample inputs with the upper bound of $\bar{e}_\zeta = 2\bar{E}_h$. To establish initial conditions, it is assumed that there exists available $N_k \in \mathbb{N}$ sample points in the prior dataset $\mathcal{D}_k = (\mathcal{G}_k, \zeta_k)$, where the sample grid $G_k = \{s(i)|i = 1, ..., N_k\} \subset \mathcal{O}$, $\mathcal{O}$ denotes the input space,

Utilizing the FLACKI rule, the prediction of $\zeta(k)$ is represented as

$$\hat{\zeta}_v(k) := \frac{1}{2}u_{k,v}(\varrho; \Xi(k)) + \frac{1}{2}l_{k,v}(\varrho; \Xi(k)), \quad (30)$$

where

$$u_{k,v}(\varrho; \theta_1(k)) := \min_i \tilde{\zeta}_v(i) + \theta_1(k)\|\varrho(k) - \varrho(s(i))\|_\infty,$$
$$l_{k,v}(\varrho; \theta_1(k)) := \max_i \tilde{\zeta}_v(i) - \theta_1(k)\|\varrho(k) - \varrho(s(i))\|_\infty,$$

$\hat{\zeta}_v(k)$ and $\tilde{\zeta}_v(i)$ denote the v-th component of the output $\hat{\zeta}(k)$ and $\tilde{\zeta}(i)$, $v = 1, 2, 3$, $i = 1, 2, ..., k-1$, $\varrho(k)$ represents the fuzzy basis function according to the following fuzzy IF-THEN inference rules:

$$\text{If} \quad s_1 \in \mathcal{S}_1^l, s_2 \in \mathcal{S}_2^l, ..., s_n \in \mathcal{S}_n^l, \quad \text{then} \quad \zeta \in \mathcal{Z}^l,$$

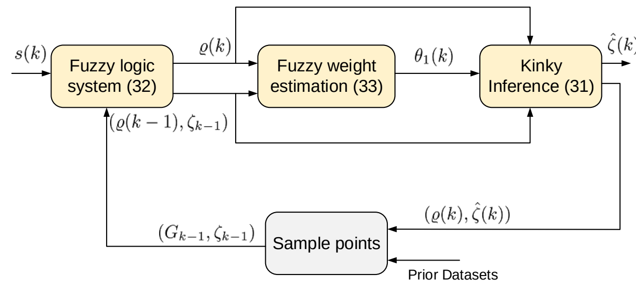

Fig. 1. Diagram of the FLACKI-based prediction.

where $s = [s_1, s_2, ..., s_n]^{\mathrm{T}} \in \mathbb{R}_n$ with $n = 3(n_u + n_\omega + 3)$ and $\zeta \in \mathbb{R}_3$ respectively express the fuzzy logic system input and output, $l = 1, 2, ..., N$ with N being the fuzzy rule numbers, $\mathcal{S}_m^l$ and $\mathcal{Z}^l$ represent the fuzzy sets, $m = 1, 2, ..., n$. Let $h_{\mathcal{Z}^l}(\zeta)$ and $h_{\mathcal{S}_m^l}(s)$ be the membership functions of $\mathcal{S}_m^l$ and $\mathcal{Z}^l$, then $\varrho(k)$ is obtained as

$$\varrho_l(k) = \frac{\prod_{i=1}^{m} h_{\mathcal{S}_i^l}(s_i)}{\sum_{l=1}^{N}\left[\prod_{i=1}^{m} h_{\mathcal{S}_i^l}(s_i)\right]}, \quad (31)$$

where $\varrho(k) = [\varrho_1(k), ..., \varrho_l(k), ..., \varrho_N(k)]$.

Let $\theta_1(k)$ denote the fuzzy weight estimation, which is obtained by the following learning rules:

$$\begin{aligned} \theta_1(k) = \max\Big\{&\theta_1(k-1), \\ &\max_{(\varrho(i),\varrho(j))\in\mathcal{V}(R_k,\{\varrho_k\})} \frac{\left\|\hat{\zeta}(i) - \hat{\zeta}(j)\right\|_\infty - \beta}{\|\varrho(i) - \varrho(j)\|_\infty}\Big\}, \end{aligned} \quad (32)$$

where the initial value of the iteration is set to be $\theta_1(0) = 0$, $R_k = R_{k-1} \cup \{\varrho_k\}$, $\mathcal{V}(R_k, \{\varrho_k\})$ is the grid of sample input pairs defined as $\mathcal{V}(S, S') := \{(s, s') \in S \times S' | \|s - s'\|_\infty > 0\}$ for two sets $S, S' \subset \mathcal{O}$, β is the hyper-parameter introduced to modify the estimation, $\beta = 2\bar{e}_\zeta + p$ with $\bar{e}_\zeta \in \mathbb{R}_{\geq 0}$ being the upper bound on the observation error of the sample sequence.

The overall diagram of the FLACKI-based prediction is given in Fig. 2, where the iterative supervised learning process is govern by (30), (31) and (32). According to **Theorem 3.1**, there is an upper bound on the prediction error under the FLACKI rule, which is expressed as:

$$\left\|\hat{\zeta}(k) - \zeta(k)\right\|_\infty \overset{k\to\infty}{\to} \left[0, \frac{p}{2} + 2\bar{e}_\zeta\right]. \quad (33)$$

IV. DATA-DRIVEN ATTITUDE CONTROL SYSTEM DESIGN

In this section, a data-driven adaptive back-stepping fault-tolerant constrained controller is designed. First, a virtual constrained control law utilizing barrier Lyapunov function is proposed for attitude kinematics loop to satisfy the deferred asymmetric performance constraint. Then, a model-free adaptive controller is designed to stabilize the angular velocity in the attitude dynamics loop. The overall control objective is to stabilize the attitude with prescribed attitude control performance when the attitude control system is subjected to actuator faults and unknown nonlinear uncertainties.

A. Deferred Asymmetric Constrained Controller Design

In the attitude kinematics loop, the following deferred asymmetric performance constraints to the attitude tracking error $q_{ev}(k) = [q_{ev1}(k), q_{ev2}(k), q_{ev3}(k)]^{\mathrm{T}} = q(k) - q_d(k)$ are taken into account:

1) The state $q(k)$ is free from constraints during the initial iteration steps $k \in [0, K_s)$.
2) The state $q(k)$ is fully constrained after iteration step K_s, that is, $q(k) \in \left(-\underline{\eta}(k), \bar{\eta}(k)\right)$ for $k \in [K_s, +\infty]$, where $-\underline{\eta}(k), \bar{\eta}(k)$ are the time-varying lower and upper bounds of $q(k)$.

In view of above constraints, the following switching function is designed:

$$\vartheta(k) = \begin{cases} 1 - \left(\dfrac{K_s - k}{K_s}\right)^3, 0 \le k < K_s \\ 1, k \ge K_s \end{cases}, \tag{34}$$

where $K_s > 0$ is the pre-defined switching time indicating the introduction of attitude constraint.

Moreover, the following properties associated with $\vartheta(k)$ are addressed:

1) $\vartheta(k)$ is strictly increasing during the initial period $k \in [0, K_s)$ with $\vartheta(0) = 0, \vartheta(K_s) = 1$. For $k \ge K_s$, $\vartheta(k)$ remains 1 as its maximum value.
2) $\vartheta(k)$ and $\Delta\vartheta(k) = \vartheta(k+1) - \vartheta(k)$ denoting the differential of $\vartheta(k)$ according to the definition of general gradient [35], are continuous and bounded for all $k > 0$.

The proof of the above properties can be found in [32].

Then, to transform the attitude response, we define that

$$\psi(k) = \vartheta(k)\, q_{ev}(k), \tag{35}$$

where $\psi(k)$ denotes the transformed attitude tracking error with consideration of omitting the initial conditions since $\psi(0) = 0$ and $\psi(K_s) = q_{ev}(K_s)$.

Next, considering the attitude control problem with deferred asymmetric constraints, the following barrier Lyapunov function based on the transformed attitude tracking error $\psi(k)$ is utilized to guarantee the attitude error performance bound, which is given as

$$V_1(k) = \frac{\psi^{\mathrm{T}}(k)\,\psi(k)}{\left(H_1(k) + \psi(k)\right)^{\mathrm{T}}\left(H_2(k) - \psi(k)\right)}, \tag{36}$$

where $H_1(k)$ and $H_2(k)$ are the barrier functions to be settled. As $q(k)$ is fully constrained by $q(k) \in \left(-\underline{\eta}(k), \bar{\eta}(k)\right)$, the barrier functions are defined as

$$H_1(k) = \underline{\eta}(k) - \underline{q}_d(k), H_2(k) = \bar{\eta}(k) - \bar{q}_d(k), \tag{37}$$

where $\underline{q}_d(k)$ and $\bar{q}_d(k)$ are continuous functions with $\underline{q}_d(k) \le q_d(k) \le \bar{q}_d(k)$.

The overall control framework is established via backstepping approach, and two dynamic surfaces are introduced to derive the overall controller. The first dynamic surface is selected as

$$S_1(k) = q_{ev}(k), \tag{38}$$

which is used to design a virtual control law for the attitude kinematics loop.

Then, the following dynamics in terms of S_1 is given as

$$\begin{aligned} S_1(k+1) &= S_1(k) + \Omega(k)\,\omega(k) - q_d(k+1) + q_d(k) \\ &= S_1(k) + \Omega(k)\left(\omega_{d_0}(k) + S_2(k)\right) - q_d(k+1) + q_d(k), \end{aligned} \tag{39}$$

where $S_2(k)$ is the second dynamic surface defined as

$$S_2(k) = \omega(k) - \omega_{d_0}(k). \tag{40}$$

Taking into consideration the transformed attitude tracking error (35), (39) is rewritten as

$$\begin{aligned} &\psi(k+1) = \psi(k) + \Delta\vartheta(k)\, S_1(k) - \vartheta(k)\,\Delta q_d(k) \\ &+ \vartheta(k)\,\Omega(k)\,\omega_{d_0}(k) + \vartheta(k)\,\Omega(k)\, S_2(k), \end{aligned} \tag{41}$$

where $\Delta\vartheta(k) = \vartheta(k+1) - \vartheta(k)$, $\Delta q_d(k) = q_d(k+1) - q_d(k)$. Assume that $q_d(k)$ is a constant vector, $\Delta q_d(k) = 0$. By introducing the definition of general gradient [35], the differential of $V_1(k)$ is derived as

$$\begin{aligned} &\Delta V_1(k) = V_1(k+1) - V_1(k) = S_1^{\mathrm{T}}(k)\left(S_1(k+1) - S_1(k)\right) \\ &= M_1(k)\,\vartheta^2(k)\, S_1^{\mathrm{T}}(k)\left(\Omega(k)\,\omega_{d_0}(k) + \Omega(k)\, S_2(k)\right. \\ &\left.+ \eta_1 S_1(k)\right) + M_1(k)\,\vartheta(k)\,\Delta\vartheta(k)\, S_1^{\mathrm{T}}(k)\, S_1(k), \end{aligned} \tag{42}$$

where

$$M_1(k) = \frac{2H_1^{\mathrm{T}}(k)\,H_2(k) - \vartheta(k)\left(H_1^{\mathrm{T}}(k) - H_2^{\mathrm{T}}(k)\right)S_1(k)}{\left(\left(H_1(k) + \vartheta(k)\,S_1(k)\right)^{\mathrm{T}}\left(H_2(k) - \vartheta(k)\,S_1(k)\right)\right)^2},$$

$$\eta_1(k) = \frac{-\Delta H_{12}(k) + \vartheta(k)\left(\Delta H_1^{\mathrm{T}}(k) - \Delta H_2^{\mathrm{T}}(k)\right)S_1(k)}{2H_1^{\mathrm{T}}(k)\,H_2(k) - \vartheta(k)\left(H_1^{\mathrm{T}}(k) - H_2^{\mathrm{T}}(k)\right)S_1(k)},$$

with

$$\begin{aligned} \Delta H_{12}(k) &= \Delta H_1^{\mathrm{T}}(k)\,H_2(k) + \Delta H_2^{\mathrm{T}}(k)\,H_1(k), \\ \Delta H_1(k) &= H_1(k+1) - H_1(k), \\ \Delta H_2(k) &= H_2(k+1) - H_2(k). \end{aligned}$$

Utilizing the famous Young's inequalities [34], the first and the last terms on the right of (42) satisfy the following inequalities:

$$\begin{aligned} &M_1(k)\,\vartheta^2(k)\,S_1^{\mathrm{T}}(k)\,\Omega(k)\,S_2(k) \le \frac{1}{4\sigma_1} S_2(k)^{\mathrm{T}} S_2(k) \\ &+ \sigma_1 M_1^2(k)\,\vartheta^4(k)\,S_1^{\mathrm{T}}(k)\,\Omega(k)\,\Omega^{\mathrm{T}}(k)\,S_1(k), \end{aligned} \tag{43a}$$

$$\begin{aligned} &M_1(k)\,\vartheta(k)\,\Delta\vartheta(k)\,S_1^{\mathrm{T}}(k)\,S_1(k) \le \frac{1}{4\sigma_2} \\ &+ \sigma_2 M_1^2(k)\,\vartheta^2(k)\,\Delta\vartheta^2(k)\,S_1^{\mathrm{T}}(k)\,S_1(k)\,S_1^{\mathrm{T}}(k)\,S_1(k), \end{aligned} \tag{43b}$$

where $\sigma_1, \sigma_2 > 0$ are tuning parameters.

Now, it is ready to construct the virtual control law as

$$\begin{aligned}&\omega_{d_0}(k) = -\Omega^{-1}(k) K_1 S_1(k) - \eta_1 \Omega^{-1}(k) S_1(k)\\&- \sigma_2 M_1(k) \Delta\vartheta^2(k) \Omega^{-1}(k) S_1(k) S_1^{\mathrm{T}}(k) S_1(k)\\&- \sigma_1 M_1(k) \vartheta^2(k) \Omega_1^{\mathrm{T}}(k) S_1(k),\end{aligned} \quad (44)$$

where $K_1 > 0$ is a feedback gain. Note that the fictitious control law (44) is introduced to the attitude dynamics loop as the reference command, and the above control law can also be seen as an attitude guidance law in practical engineering. By substituting (43a), (43b) and (44) into (42), $\Delta V_1(k)$ is obtained as

$$\begin{aligned}&\Delta V_1(k) \le -M_1(k) \vartheta^2(k) K_1 S_1^{\mathrm{T}}(k) S_1(k)\\&+ \frac{1}{4\sigma_1} S_2^{\mathrm{T}}(k) S_2(k) + \frac{1}{4\sigma_2}.\end{aligned} \quad (45)$$

Since the dynamic surface $S_2(k)$ is contained in $\Delta V_1(k)$, a data-driven approach is further introduced to derive the control input for the attitude dynamics loop in terms of the second dynamic surface $S_2(k)$ in the next subsection.

Remark 4.1: In this work, we consider the transient and steady-state performance constraints of the attitude tracking error for the entire iteration steps, and the convergence rate constraint of the attitude tracking error is also characterized. The selection principle of $H_i(k)$ mainly refers to [29], which aims to achieve fault-tolerant attitude control for spacecraft reorientation. Moreover, according to [29] and [31], the initial values of the deferred safety limitations $H_i(K_s)$ are required to comply with the constraint $H_1(K_s) > q_e v(K_s)$ or $H_2(K_s) > q_e v(K_s)$, since $H_i(K_s)$ denotes the safety constraints. This means that the attitude responses are required to maintain within boundaries defined by barrier functions at the initial step k_s. As a result, the attitude tracking error is considered to satisfy the pre-defined attitude limitations, which will not equals to or exceeds performance boundaries at $k = K_s$.

B. Model-Free Adaptive Controller Design

Considering the attitude dynamic system in (11), a model-free fault-tolerant controller is designed for the attitude dynamics loop by taking advantages of the proposed FLACKI-based predictor and adaptive control technique. In view of the second dynamic surface $S_2(k) = \omega(k) - \omega_{d_0}(k)$, the attitude dynamics in terms of S_2 is denoted as:

$$\begin{aligned}&S_2(k+1) = \omega(k+1) - \omega_{d_0}(k+1)\\&= S_2(k) + \Theta^{\mathrm{T}}(k) \Delta u_c(k) - \omega_{d_0}(k+1)\\&+ \omega_{d_0}(k) + \zeta(k).\end{aligned} \quad (46)$$

The data-driven adaptive control law for the attitude dynamics loop is designed as:

$$\begin{cases} v(k) \\ = \dfrac{-S_2(k) - \hat{\zeta}(k) - \omega_{d_0}(k+1) + \omega_{d_0}(k) + K_2 S_2(k)}{\lambda + \left\| \hat{\Theta}^{\mathrm{T}}(k) \right\|} \\ u_{c_i}(k) \\ = \begin{cases} u_{c_i}(k-1) + \rho_u \operatorname{sgn}(v_i(k)), |v_i(k)| > \rho_u \wedge |u_{ci}(k)| < \rho_{u_c} \\ u_{c_i}(k-1) + v_i(k), |v_i(k)| \le \rho_u \wedge |u_{ci}(k)| < \rho_{u_c} \\ \rho_{u_c} \operatorname{sgn}(u_{ci}(k)), |u_{ci}(k)| \ge \rho_{u_c} \end{cases} \end{cases}, \quad (47)$$

where $K_2 > 0$ and $\lambda > 0$ are parameters to be designed, $v(k) = [v_1(k), ..., v_m(k)]^{\mathrm{T}}, u_c(k) = [u_{c_1}(k), ..., u_{c_m}(k)]^{\mathrm{T}}$, $i = 1, 2, ..., m$, $\hat{\Theta}_1^{\mathrm{T}}(k)$ denotes the prediction of $\Theta_1^{\mathrm{T}}(k)$.

To obtain $\hat{\Theta}_1^{\mathrm{T}}(k)$, a cost function utilizing closed-loop data is formulated as:

$$\begin{aligned}&J\left(\Theta_1^{\mathrm{T}}(k)\right) = \left\|\omega(k) - \omega(k-1) - \Theta_1^{\mathrm{T}}(k) \Delta u_c(k-1)\right\|^2\\&+ \mu \left\|\Theta_1^{\mathrm{T}}(k) - \hat{\Theta}_1^{\mathrm{T}}(k-1)\right\|^2,\end{aligned} \quad (48)$$

where $\mu > 0$ represents the weighting coefficient. The adaptive estimation law of $\Theta^{\mathrm{T}}(k)$ can be obtained by minimizing $J\left(\Theta^{\mathrm{T}}(k)\right)$, which yields

$$\begin{aligned}&\hat{\Theta}_1^{\mathrm{T}}(k) = \hat{\Theta}_1^{\mathrm{T}}(k-1)\\&+ \frac{\gamma \left(\Delta\omega(k) - \hat{\Theta}_1^{\mathrm{T}}(k-1) \Delta u_c(k-1)\right) \Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2},\end{aligned} \quad (49)$$

where $\gamma > 0$ is the adaptive gain.

C. Convergence Analysis

Combining the contraction mapping principle and discrete-time Lyapunov theory, the convergence of all the dynamic surfaces and estimation error of $\tilde{\Theta}_1^{\mathrm{T}}$ is analyzed. Before details, the lemma of contraction mapping principle is given.

Lemma 4.1 [11]: Consider the iteration of state $\nu(k)$, which is expressed as

$$\nu(k+1) = \alpha(k) \nu(k) + \varpi(k), \quad (50)$$

where $\alpha(k) \in \mathbb{R}_{M \times M}$ denotes the mapping matrix and $\varpi(k) \in \mathbb{R}_M$ is the control input. If $\|\alpha(k)\| < 1$, $\lim_{k\to\infty} \varpi(k) = 0$, then $\lim_{k\to\infty} \nu(k) = 0$.

According to **Lemma 4.1**, we can obtain the following theorem.

Theorem 4.1: For the discrete nonlinear attitude control system (11) and the adaptive law (49), if $0 < \gamma < 2$, the boundedness of estimation error of $\hat{\Theta}_1^{\mathrm{T}}$ can be guaranteed, and the upper bound of the error can be derived as

$$\lim_{k\to\infty} \left\|\tilde{\Theta}_i^{\mathrm{T}}(k)\right\| = \frac{a_1}{1-a}, \quad (51)$$

where $\tilde{\Theta}_i^{\mathrm{T}}(k) = \Theta_i^{\mathrm{T}}(k) - \hat{\Theta}_i^{\mathrm{T}}(k)$ is the adaptive estimation error of $\hat{\Theta}_i^{\mathrm{T}}(k)$, $\hat{\Theta}_i^{\mathrm{T}}(k)$ is the estimation of $\Theta_i^{\mathrm{T}}(k)$

obtained by (49), $a = \left\| I - \frac{\gamma \Delta u_c(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} \right\|$, $a_1 = \frac{\gamma(\bar{\zeta}_l + \bar{\zeta}_b)}{2\sqrt{\mu}} + 2\rho_\Theta$, $\Theta_i^{\mathrm{T}}(k) \in \mathbb{R}^{1\times m}$ is the i-th row of $\Theta_1^{\mathrm{T}}(k)$.

Proof: Recall the adaptive law (49), the estimation error can be denoted as

$$\begin{aligned}
\tilde{\Theta}_i^{\mathrm{T}}(k) &= \tilde{\Theta}_i^{\mathrm{T}}(k-1) + \Delta\Theta_i^{\mathrm{T}}(k) - \frac{\gamma}{\mu + \|\Delta u_c(k-1)\|^2} \\
&\times \Big(\Theta_i^{\mathrm{T}}(k-1)\Delta u_c(k-1) + \zeta_i(k-1) \\
&- \hat{\Theta}_i^{\mathrm{T}}(k-1)\Delta u_c(k-1) \Big) \Delta u_c^{\mathrm{T}}(k-1) \\
&= \tilde{\Theta}_i^{\mathrm{T}}(k-1)\left(I - \frac{\gamma \Delta u_c(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} \right) \\
&- \frac{\gamma \zeta_i(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} + \Delta\Theta_i^{\mathrm{T}}(k), \quad (52)
\end{aligned}$$

where $\zeta_i(k-1)$ is the $(k-1)$-th iteration of the i-th row of the uncertainties ζ, $\Delta\Theta_i^{\mathrm{T}}(k) = \Theta_i^{\mathrm{T}}(k) - \Theta_i^{\mathrm{T}}(k-1)$. According to the boundedness assumption of $\Theta^{\mathrm{T}}(k)$, the upper bound of partial terms in (52) can be obtained as

$$\left\| \frac{\gamma \zeta_i(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} - \Delta\Theta_i^{\mathrm{T}}(k) \right\| \le \frac{\gamma \bar{\zeta}_l}{2\sqrt{\mu}} + 2\rho_\Theta = b. \quad (53)$$

Taking norm on both sides of (53), it can be derived as

$$\begin{aligned}
&\|\tilde{\Theta}_i^{\mathrm{T}}(k)\| \\
&= \left\| \tilde{\Theta}_i^{\mathrm{T}}(k-1)\left(I - \frac{\gamma \Delta u_c(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} \right) \right\| + a_1. \quad (54)
\end{aligned}$$

Note that

$$\begin{aligned}
&\left\| \tilde{\Theta}_i^{\mathrm{T}}(k-1)\left(I - \frac{\gamma \Delta u_c(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} \right) \right\| \\
&\le \|\tilde{\Theta}_i^{\mathrm{T}}(k-1)\| \left\| I - \frac{\gamma \Delta u_c(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} \right\|, \quad (55)
\end{aligned}$$

since $0 < \gamma < 2$, $\mu > 0$, then

$$0 < \left\| I - \frac{\gamma \Delta u_c(k-1)\Delta u_c^{\mathrm{T}}(k-1)}{\mu + \|\Delta u_c(k-1)\|^2} \right\| = a < 1$$

is satisfied, and one can obtain

$$\begin{aligned}
\|\tilde{\Theta}_i^{\mathrm{T}}(k)\| &\le a \|\tilde{\Theta}_i^{\mathrm{T}}(k-1)\| + a_1 \\
&\le a^k \|\tilde{\Theta}_i^{\mathrm{T}}(0)\| + \frac{a_1}{1-a}. \quad (56)
\end{aligned}$$

Form **lemma 4.1**, the boundedness of $\tilde{\Theta}_i^{\mathrm{T}}(k)$ can be obtained with the upper bound expressed as $\lim_{k\to\infty} \|\tilde{\Theta}_i^{\mathrm{T}}(k)\| = \frac{a_1}{1-a}$. ■

To this end, the main result of this paper is summarized in the following theorem, and the control framework is shown in Fig. 2.

Theorem 4.2: Under the **Assumption 2.1** to **Assumption 2.4**, consider the nonlinear discrete attitude kinematics and dynamics in (11) with the data-driven controllers (44) and (47), the adaptive estimation law (49) and FLACKI-based predictions (30) and (32), the closed-loop system is semi-globally uniform ultimate bounded and the attitude tracking error satisfies the deferred asymmetric prescribed constraints, if the control and estimation gains are selected to satisfy the following selection principles:

$$K_1 > 0,\ K_2\Gamma_1 - 2\Gamma_1 - \frac{I}{4\sigma_1} - 3I - \frac{\kappa^2 I}{\gamma_2} < 0, \quad (57)$$

where $\Gamma_1 = \frac{H(k)\Theta^{\mathrm{T}}(k)}{\lambda + \|\hat{\Theta}^{\mathrm{T}}(k)\|}$, γ_2 is a positive tuning parameter, $H(k) = \frac{\Delta u_c(k)}{v(k)}$.

Proof: The overall Lyapunov function is chosen as

$$V(k) = V_1(k) + \frac{1}{2} S_2^{\mathrm{T}}(k) S_2(k). \quad (58)$$

The differential of $V(k)$ can be approximately described as

$$\Delta V(k) = \Delta V_1(k) + S_2^{\mathrm{T}}(k)(S_2(k+1) - S_2(k)). \quad (59)$$

In the view of (45), $\Delta V(k)$ is expressed as

$$\begin{aligned}
&\Delta V(k) \le -M_1(k)\vartheta^2(k) K_1 S_1^{\mathrm{T}}(k) S_1(k) \\
&+ \frac{1}{4\sigma_1} S_2^{\mathrm{T}}(k) S_2(k) + \frac{1}{4\sigma_2} + S_2^{\mathrm{T}}(k)(S_2(k+1) - S_2(k)). \quad (60)
\end{aligned}$$

Define $\Delta V_2(k) = S_2^{\mathrm{T}}(k)(S_2(k+1) - S_2(k))$, utilizing $H(k) = \frac{\Delta u_c(k)}{v(k)}$, note that $\|\Delta u_c(k)\| \le \|v(k)\|$ from (47), then $0 < H(k) \le 1$, $\Delta V_2(k)$ can be rewritten as

$$\begin{aligned}
&\Delta V_2(k) = S_2^{\mathrm{T}}(k)\big(S_2(k) + \Theta^{\mathrm{T}}(k)\Delta u_c(k) \\
&-\omega_{d_0}(k+1) + \omega_{d_0}(k) + \zeta(k)\big) \\
&= S_2^{\mathrm{T}}(k)\left(\tilde{\zeta}(k) + \left(I - \frac{H(k)\Theta^{\mathrm{T}}(k)}{\lambda + \|\hat{\Theta}^{\mathrm{T}}(k)\|} \right) \Big(S_2(k) + \hat{\zeta}(k) \right. \\
&\left. -\omega_{d_0}(k+1) + \omega_{d_0}(k) \Big) + \frac{K_2 H(k)\Theta^{\mathrm{T}}(k)}{\lambda + \|\hat{\Theta}^{\mathrm{T}}(k)\|} S_2(k) \right). \quad (61)
\end{aligned}$$

Denoting $\Gamma_1 = \frac{H(k)\Theta^{\mathrm{T}}(k)}{\lambda + \|\hat{\Theta}^{\mathrm{T}}(k)\|}$ and utilizing Young's inequality, $\Delta V_2(k)$ is derived as

$$\begin{aligned}
&\Delta V_2(k) = (I - (I - K_2)\Gamma_1) S_2^{\mathrm{T}}(k) S_2(k) + S_2^{\mathrm{T}}(k)\tilde{\zeta}(k) \\
&+ (I - \Gamma_1) S_2^{\mathrm{T}}(k)\left(-\omega_{d_0}(k+1) + \omega_{d_0}(k) + \hat{\zeta}(k) \right) \\
&\le (3I - (2I - K_2)\Gamma_1) S_2^{\mathrm{T}}(k) S_2(k) + \frac{1}{4}\hat{\zeta}^{\mathrm{T}}(k)\hat{\zeta}(k) \\
&+ (I - \Gamma_1) S_2^{\mathrm{T}}(k)\Delta\omega_{d_0}(k) + \frac{1}{4}\tilde{\zeta}^{\mathrm{T}}(k)\tilde{\zeta}(k), \quad (62)
\end{aligned}$$

where $\Delta\omega_{d_0} = \omega_{d_0}(k+1) - \omega_{d_0}(k)$. Then, according to (44), it follows that

$$\|\Delta\omega_{d_0}\| = \|\omega_{d_0}(k+1) - \omega_{d_0}(k)\| \le \chi(S_1, S_2, q_d), \quad (63)$$

where $\chi(\cdot)$ is a scalar continuous function. The following inequalities can be obtained as

$$\begin{aligned}
&(I - \Gamma_1) S_2^{\mathrm{T}}(k)\Delta\omega_{d_0}(k) \\
&\le \frac{(I - \Gamma_1)\|\chi\|^2}{2\gamma_2} S_2^{\mathrm{T}}(k) S_2(k) + \frac{\gamma_2}{2}. \quad (64)
\end{aligned}$$

Consider a compact set $\Psi := \{(S_1, S_2, q_d) : V < b\}$, the maximum value κ of $\|\chi\|$ exists on Ψ, synthesizing (62)

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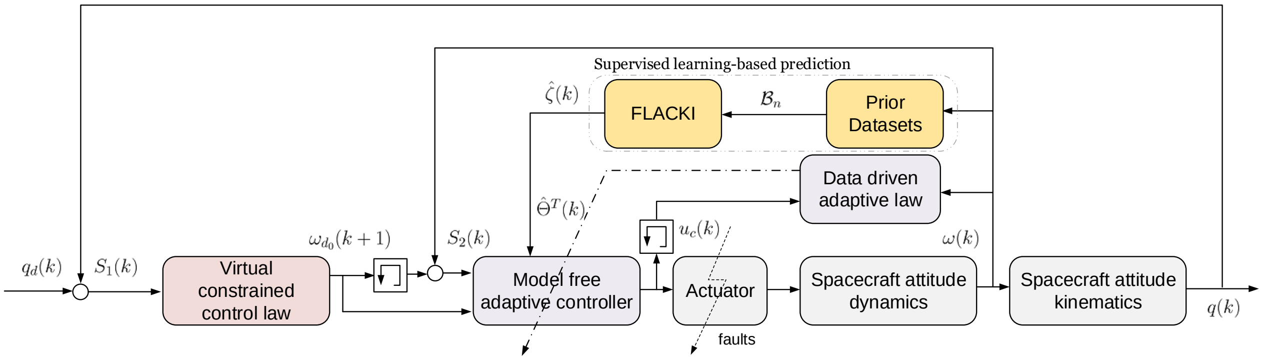

Fig. 2. Control Framework.

and (64), $\Delta V_2(k)$ is denoted as

$$\begin{aligned}&\Delta V_2(k)\\ &\le\left(3I-(2I-K_2)\Gamma_1+\frac{\kappa^2(I-\Gamma_1)}{2\gamma_2}\right)S_2^{\mathrm T}(k)S_2(k)\\ &+\frac{\gamma_2}{2}+\frac{1}{4}\hat\zeta^{\mathrm T}(k)\hat\zeta(k)+\frac14\tilde\zeta^{\mathrm T}(k)\tilde\zeta(k).\end{aligned}\tag{65}$$

From the boundedness of $\tilde\zeta(k),\zeta(k)$, note that $0<\|\Gamma_1\|<1$, one can obtain

$$\begin{aligned}&\Delta V(k)=-M_1(k)\vartheta^2(k)K_1S_1^{\mathrm T}(k)S_1(k)\\ &+\frac{1}{4\sigma_2}+\frac{\gamma_2}{2}+\frac14\left\|\hat\zeta(k)\right\|^2+\frac14\|\tilde\zeta(k)\|^2\\ &+\left(\frac{I}{4\sigma_1}+3I+(K_2-2I)\Gamma_1+\frac{\kappa^2(I-\Gamma_1)}{2\gamma_2}\right)S_2^{\mathrm T}(k)S_2(k)\\ &\le-M_1(k)\vartheta^2(k)K_1S_1^{\mathrm T}(k)S_1(k)\\ &+\frac{1}{4\sigma_2}+\frac{\gamma_2}{2}+\frac14\bar\zeta_l^2+\frac14\left(\frac p2+2\bar e_\zeta\right)^2\\ &+\left(\frac{I}{4\sigma_1}+3I-2\Gamma_1+K_2\Gamma_1+\frac{\kappa^2I}{\gamma_2}\right)S_2^{\mathrm T}(k)S_2(k).\end{aligned}\tag{66}$$

Then, if parameters are selected such that the conditions in (57) hold, then $\Delta V(k)$ can satisfy the following inequality on $V=b$:

$$\Delta V(k)\le-\gamma_0b+\beta_0,\tag{67}$$

where $\gamma_0=\min\{M_1(k)\vartheta^2(k)K_1,2\Gamma_1-\frac{I}{4\sigma_1}-3I-K_2\Gamma_1-\frac{\kappa^2I}{\gamma_2}\}$, $\gamma_0>0$, $\beta_0=\frac{1}{4\sigma_2}+\frac{\gamma_2}{2}+\frac14\bar\zeta_l^2+\frac12\left(\frac p2+2\bar e_\zeta\right)^2$. Furthermore, integrating (66) yields

$$0\le V(k)\le\frac{\beta_0}{\gamma_0}+\left(V(0)-\frac{\beta_0}{\gamma_0}\right)\mathrm e^{-\gamma_0k}.\tag{68}$$

That is, $V(k)\in L_\infty$, as long as $\gamma_0>\frac{\beta_0}{b}$, $\Delta V(k)\le 0$, $V(k)\le b$ is an invariant set, which indicates the boundedness of the asymmetric barrier Lyapunov function $V_1(k)$, implying that $\psi(k)$ satisfies $-H_1(k)\le\psi(k)\le H_2(k)$. In addition, in view of $\vartheta(0)=0$ for $k<K_s$, $\psi(k)=\vartheta(k)S_1(k)=0$. That is, the constraint of the attitude response is relaxed before iteration step K_s. After the switching instant K_s, due to $\vartheta(k)=1$, it is clear that $\psi(k)=S_1(k)$ and $-H_1(k)\le S_1(k)\le H_2(k)$, which realizes the asymmetric prescribed performance of attitude tracking error. Then, according to (67), it can be derived that

$$\begin{aligned}&\frac{\psi^{\mathrm T}(k)\psi(k)}{(H_1(k)+\psi(k))^{\mathrm T}(H_2(k)-\psi(k))}=V_1(k)\le V(k)\\ &\le\frac{\beta_0}{\gamma_0}+\left(V(0)-\frac{\beta_0}{\gamma_0}\right)\mathrm e^{-\gamma_0k}.\end{aligned}\tag{69}$$

Due to Young's inequality [25], in the compact set $-H_1(k)\le\psi(k)\le H_2(k)$, $(H_1(k)+\psi(k))^{\mathrm T}(H_2(k)-\psi(k))\le(H_1(k)+H_2(k))^{\mathrm T}(H_1(k)+H_2(k))/4$, and $\|\psi(k)\|\le\frac{H_1(k)+H_2(k)}{2}\sqrt{\frac{\beta_0}{\gamma_0}+\left(V(0)-\frac{\beta_0}{\gamma_0}\right)\mathrm e^{-\gamma_0k}}$ is obtained. As the iteration step $k\to\infty$, it is derived that

$$\lim_{k\to\infty}\|\psi(k)\|=\lim_{k\to\infty}\|S_1(k)\|=\frac{H_1(k)+H_2(k)}{2}\sqrt{\frac{\beta_0}{\gamma_0}},\tag{70}$$

which means the steady response of attitude tracking error can be bounded and becomes smaller by choosing γ_0 properly.

Recall the derivation result that $V(k)\le b$ is an invariant set when $\gamma_0>\frac{\beta_0}{b}$, $\Delta V(k)\le 0$ can be guaranteed by satisfying the parameter selection principle. It can be concluded that the system is semi-globally uniform ultimate bounded meanwhile attitude tracking error can coverage into the asymmetric bound $[-H_1(k),H_2(k)]$ by the barrier Lyapunov function-based control scheme. This completes the proof. ■

V. SIMULATION RESULTS AND ANALYSIS

A. Parameter Settings

Numerical simulations are provided to demonstrate the effectiveness of the FLACKI-based data-driven adaptive back-stepping fault-tolerant control scheme. The objective of this simulation is to reorientate the spacecraft to the desired attitude $q_d=[0,0,0,1]^{\mathrm T}$ by using the proposed data-driven control scheme, with the consideration of deferred asymmetric constraints in attitude response, time-varying actuator faults, and modeling uncertainties. In the numerical simulation, the sampling frequency is 10 Hz,

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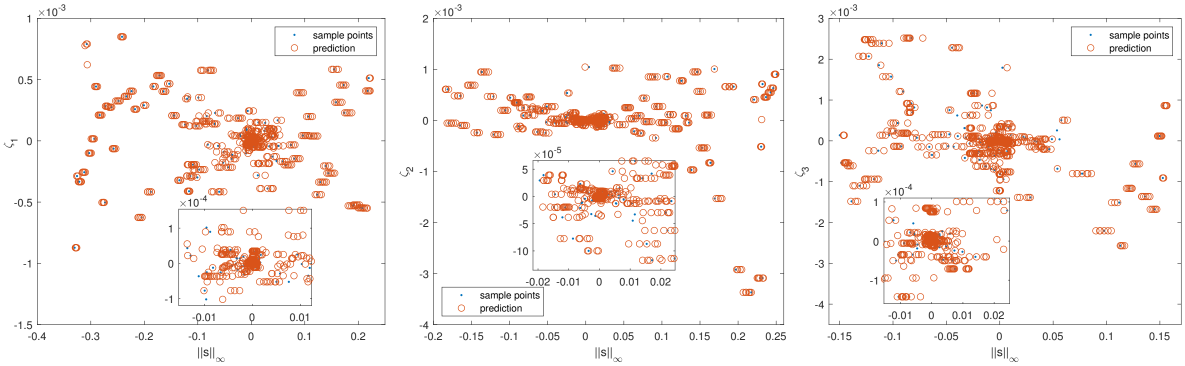

Fig. 3. Prediction by FLACKI rule.

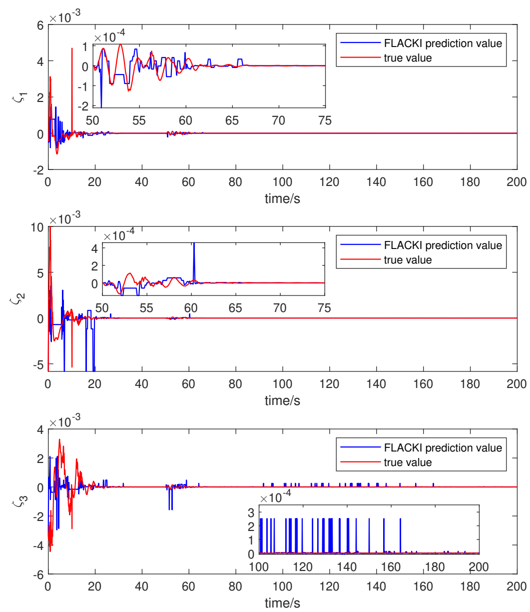

Fig. 4. Comparison of FLACKI prediction and true value.

that is, the sample step satisfies $k = 10t$, where t is the simulation time. Parameters of the spacecraft dynamics are given as follows [16]:

$$J_0 = \begin{bmatrix} 30 & 1 & 0.5 \\ 1 & 20 & 3 \\ 0.5 & 3 & 10 \end{bmatrix} \text{kg}\cdot\text{m}^2$$

with the initial angular velocity and attitude selected as $\omega_0 = [0.06, -0.05, -0.1]^{\mathrm{T}}$ rad/s and $q_0 = [0.4, -0.3, -0.4, 0.7681]^{\mathrm{T}}$. The upper bound of control torque and input rate are set to be $\rho_u = 1, \rho_{u_c} = 5$. The modeling uncertainty of inertia matrix is assumed to be in the form of $\Delta J = -J_1\psi(t) + \tilde{J}$ [33], where $J_1\psi(t)$ is

$$J_1\psi(t) = -[m_1 I_3, m_2 I_3] \begin{bmatrix} \eta_1^{\mathrm{T}}\eta_1 I_3 - \eta_1\eta_1^{\mathrm{T}} \\ \eta_2^{\mathrm{T}}\eta_2 I_3 - \eta_2\eta_2^{\mathrm{T}} \end{bmatrix}, \tag{71}$$

with time-varying coefficients η_1 and η_2 defined as

$$\eta_1(t) = 0.1\left[1.2\kappa_1 t + 0.05\sin(3t) + +0.05\cos(3t)\right],$$
$$\eta_2(t) = -0.1\left[1.2\kappa_1 t + 0.05\sin(3t) + +0.05\cos(3t)\right],$$

and κ_1 being a time-varying coefficient given by

$$\kappa_1 = \begin{cases} 1 & t \le 10 \\ \frac{10}{t} & t > 10 \end{cases}.$$

Additionally, $\tilde{J}$ is denoted as

$$\tilde{J} = \begin{bmatrix} 0 & 0.05\sin(3t) & 0.05\sin(3t) \\ 0.02\sin(3t) & 0 & 0.03\sin(3t) \\ -0.02\sin(3t) & 0.03\sin(3t) & 0 \end{bmatrix} \text{kg}\cdot\text{m}^2.$$

The external disturbance is set as:

$$d = 0.01 \times \begin{bmatrix} \sin(0.1||\omega||t) + ||\omega|| \\ \sin(0.075||\omega||t) + ||\omega|| \\ \sin(0.05||\omega||t) + ||\omega|| \end{bmatrix} \text{N}\cdot\text{m}.$$

In the simulation, a time-varying bias fault is introduced into the control input at 50 s, which is denoted as

$$\bar{u} = 0.05 \times \begin{bmatrix} 0.9||\omega|| + 0.1\sin(0.1||\omega||t) + 0.01\text{rand}(\cdot) \\ 0.9||\omega|| + 0.1\sin(||\omega||t/15) + 0.015\text{rand}(\cdot) \\ 0.9||\omega|| + 0.1\sin(0.05||\omega||t) + 0.02\text{rand}(\cdot) \end{bmatrix} \text{N}\cdot\text{m},$$

and the gain fault matrix is defined as

$$E = \begin{bmatrix} 0.89 + 0.1\sin(0.1t) + 0.01\text{rand}(\cdot) \\ 0.885 + 0.1\sin(t/15) + 0.015\text{rand}(\cdot) \\ 0.88 + 0.1\sin(0.05t) + 0.02\text{rand}(\cdot) \end{bmatrix}.$$

where the function rand$(\cdot)$ produces a random value from the normal distribution with mean 0 and standard deviation 1.

A barrier Lyapunov function is introduced to the attitude kinematics control loop, which defines the time-varying asymmetric prescribed performance constraints for the attitude response. Specifically, the asymmetric performance is given as follows:

$$\iota(k) = 0.2\mathrm{e}^{-0.004k} + 0.01,$$
$$H_1(k) = (0.6\iota(k) - 0.01)1_n,$$
$$H_2(k) = (0.9\iota(k) + 0.03)1_n,$$

where 1_n denotes an n-dimensional column vector of ones. To omit the influence of the initial value conditions, a deferred switching function is introduced to $S_1(k)$, with the pre-defined switching step set as $K_s = 500$, which means the switching time set to be 50 s.

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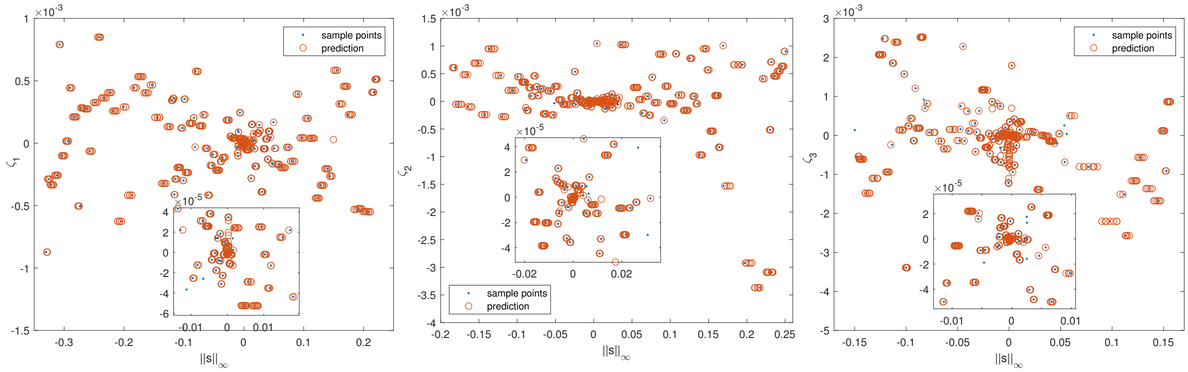

Fig. 5.   Prediction by LACKI rule.

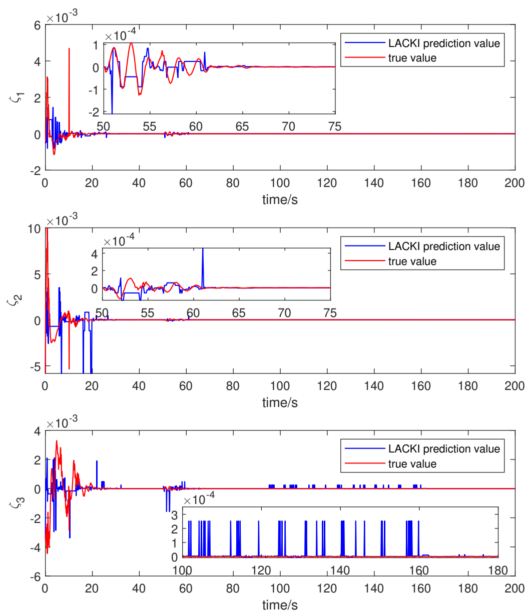

Fig. 6.   Comparison of LACKI prediction and true value.

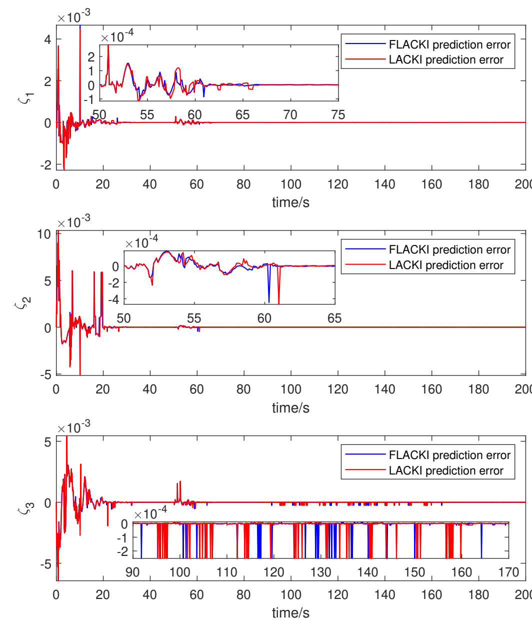

Fig. 7.   Comparison of prediction error between LACKI and FLACKI.

To verify the effectiveness of the FLACKI rule-based data-driven back-stepping adaptive fault-tolerant control scheme, comparison with existing data-driven method proposed in [11] is also simulated, in which a data-driven ADRC method is used with an extended state observer to deal with the system uncertainties. To have a fair comparison, the ADRC-based data-driven method is implemented to the inner loop with the back-stepping control framework, while the outer loop remains the same as the proposed scheme in this work. The ADRC-based controller is given as follows:

$$\begin{cases} v\left(k\right) \\ \quad = \dfrac{-\omega\left(k\right) - z_2\left(k\right) + \omega_d\left(k+1\right) + K_2\left(\omega_d\left(k\right) - z_1\left(k\right)\right)}{\lambda + \left\|\hat{\Theta}^{\mathrm{T}}\left(k\right)\right\|}, \\ u_{c_i}\left(k\right) = \begin{cases} u_{c_i}\left(k-1\right) + \rho_u \mathrm{sgn}\left(v_i\left(k\right)\right), \left|v_i\left(k\right)\right| > \rho_u, \\ u_{c_i}\left(k-1\right) + v_i\left(k\right), \left|v_i\left(k\right)\right| \le \rho_u, \end{cases} \end{cases} \tag{72}$$

where parameters $K_2 > 0$ and $\lambda > 0$ are selected the same as in the scheme proposed in the simulation. $z_1\left(k\right), z_2\left(k\right)$ are the estimation from the data-driven extended state observer, which is given by

$$\begin{cases} z_1\left(k+1\right) = z_1\left(k\right) + \hat{\Theta}^{\mathrm{T}}\left(k\right)\Delta u_c\left(k\right) \\ \quad + z_2\left(k\right) + l_1\left(\omega\left(k\right) - z_1\left(k\right)\right), \\ z_2\left(k+1\right) = z_2\left(k\right) + l_2\left(\omega\left(k\right) - z_1\left(k\right)\right), \end{cases} \tag{73}$$

where the observer gains are set as $l_1 = 9.8$ and $l_2 = 8.8$.

For the proposed virtual controller (44) of the attitude kinematics loop and the FLACKI-based data-driven adaptive controller (47), the parameters are selected as $\lambda = 0.01$, $\mu = 0.01$, $\gamma = 1$, $K_1 = 1.76$, $\sigma_1 = 0.008$, $\sigma_2 = 0.007$, $K_2 = 0.9$. In addition, in the FLACKI rule, a fuzzy logic system with 4 fuzzy rules are used and the fuzzy membership functions are set as following:

$$h_{S_i^l}\left(s_i\right) = \exp\left[-(20s_i + 0.3(l-1))^2\right],$$

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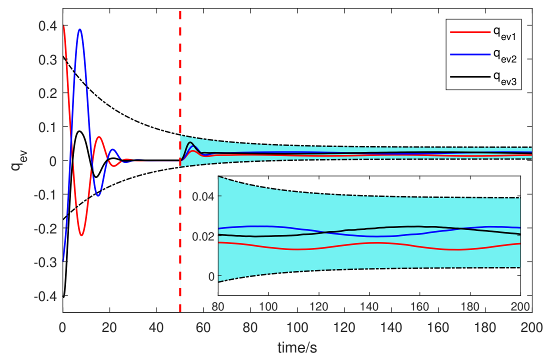

Fig. 8. Attitude response with asymmetric performance constraints.

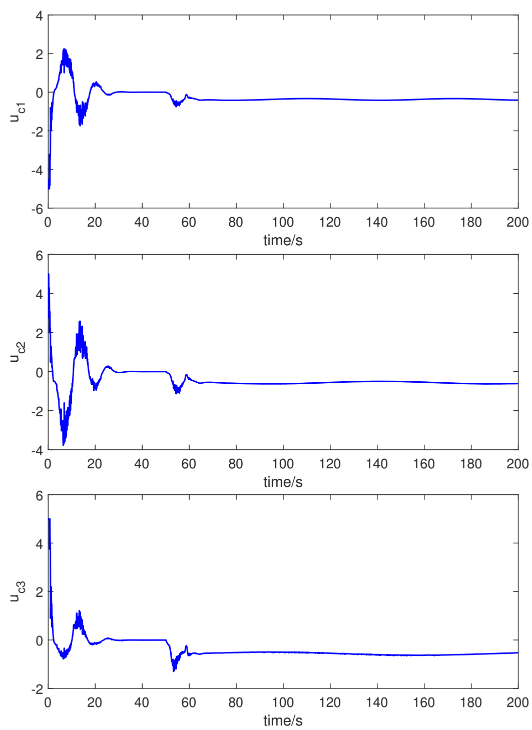

Fig. 9. Input signals.

where $i = 1, ..., n$, $n = 3(n_u + n_\omega + 3)$, $n_u = n_\omega = 1$, $l = 1, ..., 4$.

B. FLACKI Rule

To demonstrate the benefits of the proposed online FLACKI rule in (30) and (32), we first compare it with the existing LACKI rule in [23], in which 141 data points are sampled. The simulation time is 200 s, and the fixed

TABLE I
Results of FLACKI and LACKI.

Rule	Prior points	RMSE×10^{-5}			Time/s
		ζ_1	ζ_2	ζ_3	
FLACKI	141	2.78	11.2	17.6	332.07
LACKI	141	7.90	33.3	61.3	220.50
LACKI	301	2.38	17.1	48.9	467.90

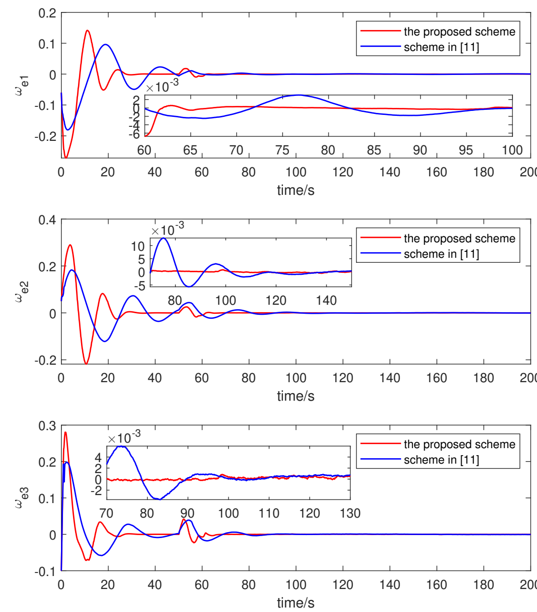

Fig. 10. Comparison of angular velocity response between the proposed scheme and the scheme in [11] in the presence of actuator faults.

sample step is set to be 0.1 s. This means that 2001 sample points are predicted by the 141 prior sample grids.

The simulation results of the proposed FLACKI rule are shown in Fig. 3 and Fig. 4, while Fig. 5 and Fig. 6 depicts the prediction result of the existing LACKI rule in [23]. Specifically, it is seen from Fig. 3 to Fig. 6 that bounded prediction of the true value of the system's unknown dynamics can be achieved by either FLACKI rule and LACKI rule. Based on the theoretical analysis, this estimation error is mainly caused by the external terms not related to the input grids. The comparison of prediction error between LACKI and FLACKI is plotted in Fig. 7, and the quantitative comparison in terms of root mean square error (RMSE) of the prediction error and simulation time under LACKI and FLACKI rules are shown in TABLE I. It can be observed that although computing time increases with the introduction of the fuzzy logic in FLACKI rule, the prediction precision is improved by 69.2%. It is well known from [23], [25] that the computing time will grow rapidly with more sample grids. To verify the above illustration, we also provide the simulation of the LACKI-based prediction with 301 initial sample points for achieving similar prediction accuracy compared with the proposed FLACKI rule. As seen in TABLE I, it is clear that more prior points and computational time are needed for the existing LACKI.

C. Data-Driven Adaptive Back-Stepping Fault-Tolerant Controller

In this subsection, we first verifies the effectiveness of the proposed data-driven adaptive back-stepping fault-

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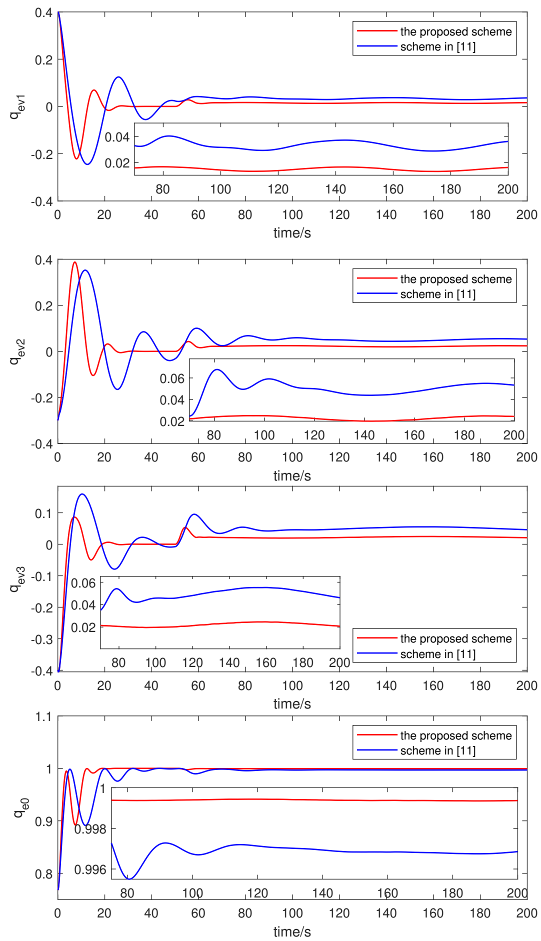

Fig. 11. Comparison of attitude response between the proposed scheme and the scheme in [11] in the presence of actuator faults.

tolerant controller in (44) and (47), then the advantages of the proposed controller are demonstrated by comparing with the existing data-driven ADRC method in [11] (cf. (72) and (73)).

As shown in Fig. 8, it is clear that the time response of attitude converges to a prescribed ultimate bound during the control progress, which has been analyzed theoretically in the previous sections. It is worth mentioning that the attitude response does not have to satisfy the time-varying deferred asymmetric constraints before the switching time. After the predefined switching time of 50 s, all the attitude error responses converge into the performance bound despite the existence of disturbance and actuator faults. Fig. 9 depicts the commanded control inputs (47) of the proposed FLACKI-based scheme.

The comparison results of the attitude and angular velocity responses under the proposed scheme the ADRC-based data-driven approach [11] are shown in Fig. 10 to Fig. 13, where the simulation conditions are respectively set in the presence of actuator faults and in the absence of actuator faults. It is observed that, in either the fault-free condition or the faulty condition, the proposed scheme achieves better uncertainty suppression performance than that of the data-driven ADRC scheme for attitude control

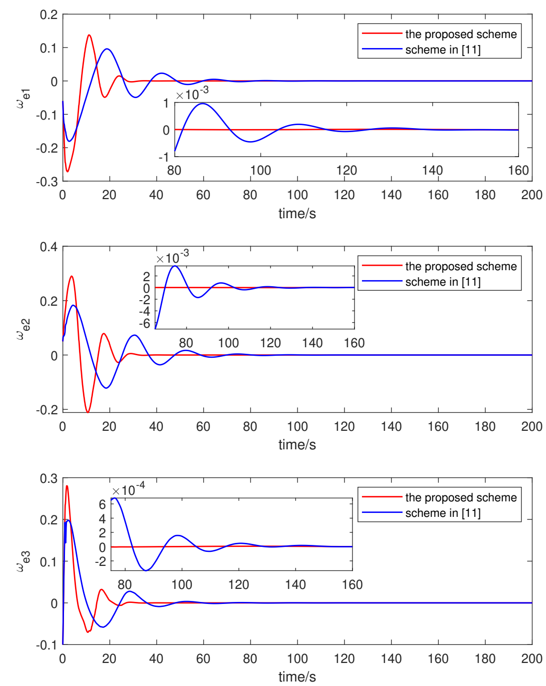

Fig. 12. Comparison of angular velocity response between the proposed scheme and the scheme in [11] in the absence of actuator faults.

of spacecraft. This is due to that estimation ability of the data-driven ADRC is mainly depended on the feedback gain chosen of the extended state observer, which is difficult to choose suitable parameters to estimate and suppress the time-varying unmodeled dynamics. As a result, the ADRC scheme cannot obtain high tracking precision and leads to large steady-state tracking error, as shown in Fig. 11. On the other hand, as long as the model uncertainties are arbitrarily continuous, the prediction error under the FLACKI rule is small, so the proposed data-driven model-free controller in (47) can achieve satisfactory control performance for spacecraft attitude control.

VI. CONCLUSION

In this paper, a data-driven adaptive back-stepping fault-tolerant control scheme is proposed for the spacecraft attitude control problem. To deal with the uncertainties, external disturbances and time-varying actuator faults, a supervised learning scheme named FLACKI rule is introduced to estimate arbitrarily continuous actuator faults and other uncertainties, resulting in bounded estimation error. The overall control scheme contains two parts, where a virtual control law derived from a barrier Lyapunov function is used to guarantee the deferred asymmetric constraints of attitude tracking error in the attitude kinematics loop and a model-free fault-tolerant controller using the estimation from FLACKI-based predictor is

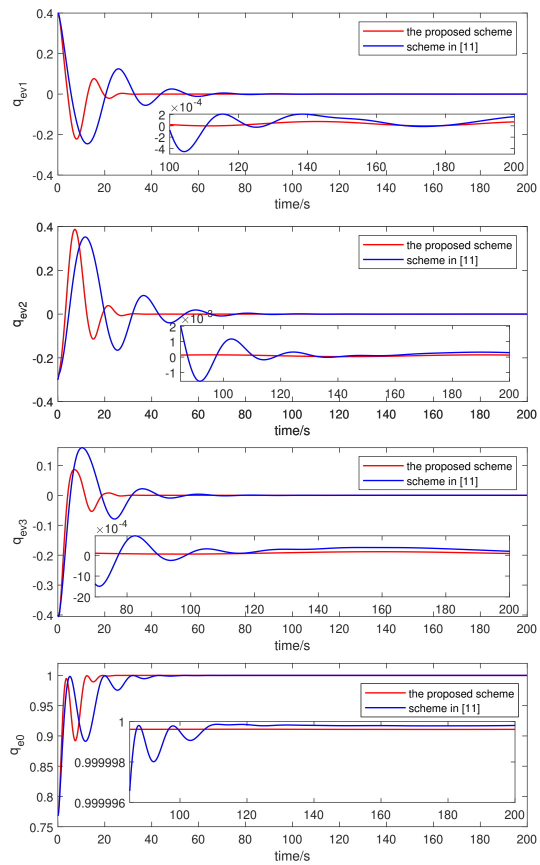

Fig. 13. Comparison of attitude response between the proposed scheme and the scheme in [11] in the absence of actuator faults.

designed to compensate unmodeled arbitrarily continuous dynamics. The stability analysis of the overall closed-loop system is performed with the aid of contraction mapping principle and discrete-time Lyapunov theory. Numerical simulation verifies the efficiency of the proposed FLACKI-based prediction and the model-free fault-tolerant control scheme. In future work, the situation that the initial attitude error exceeds the boundaries defined by the barrier function will be explored.

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