Integral-Type Sliding Mode Fault-Tolerant Control for Attitude Stabilization of Spacecraft

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***Abstract*—Two fault-tolerant control schemes for spacecraft attitude stabilization with external disturbances are proposed in this paper. The approach is based on integral-type sliding mode control strategy to compensate for actuator faults without controller reconfiguration. Firstly, a basic integral-type sliding mode fault-tolerant control scheme is designed so that sliding manifold can be maintained from the very beginning. Once the system enters the sliding mode, the dynamics of the closed-loop system with actuator fault is identical to that of the nominal healthy system. Secondly, the integral-type sliding mode fault-tolerant controller is incorporated with adaptive technique to accommodate actuator faults so that the required boundary information can be relaxed. The effectiveness of the proposed schemes against actuator faults is demonstrated in simulation.**

***Index Terms*—Fault-tolerant control (FTC), actuator fault, attitude stabilization, integral-type sliding mode control (ISMC), spacecraft.**

I. Introduction

IN recent years, due to the increasing demands for safety and reliability in modern industrial systems, especially for life-critical systems such as spacecraft, aircraft, nuclear power plant and so on, fault-tolerant control (FTC) has received considerable attention [1]–[3]. As is well known, the available FTC schemes can be generally classified into two categories, i.e. passive approach and active approach. Here, in this paper, we focus on only the passive approach and apply it to spacecraft attitude stabilization problems in the presence of actuator faults.

Some effective passive FTC approaches have been proposed for both linear systems and nonlinear systems, such as H_∞ theory [4], linear matrix inequalities (LMIs) techniques [5], Lyapunov reconstruction [6], [7], and sliding mode control (SMC) [8], [9]. Among these approaches, SMC technique is recognized as an efficient way to withstand matched external disturbances and model uncertainties, and has been widely adopted in spacecraft attitude FTC systems. In [10], sliding mode fault-tolerant controller was proposed for a set of second-order nonlinear systems under actuator faults, where the proposed fault-tolerant controller was shown to ensure local asymptotic stability when it was applied to spacecraft attitude stabilization. Based on SMC technique, a fault-tolerant control scheme for flexible spacecraft attitude stabilization system using redundant actuators was proposed in [11]. In [12], two adaptive fault-tolerant sliding mode control laws were developed for multiple spacecraft formation flying, where the global asymptotic convergence of the position tracking error was achieved in the presence of uncertain system parameters, external disturbances and actuator faults. In [13], an adaptive sliding mode fault-tolerant attitude tracking control scheme was developed for flexible spacecraft with partial loss of actuator effectiveness fault, where a neural network was employed to account for system uncertainties and an on-line updating law was used to estimate the upper bound of actuator fault.

The aforementioned classical SMC-based fault-tolerant approaches, although robust against matched actuator faults and uncertainties, have some disadvantages. It was reported that the system dynamics might be vulnerable to faults or uncertainties during the reaching phase in which the system states have not yet reached the sliding manifold [14]. With a view to tackling the reaching phase problem, the concept of integral-type SMC (ISMC) was proposed in [15]–[18]. The basic idea of ISMC is to design a proper sliding manifold such that the sliding mode starts from the initial time instant. As a result, the robustness of the system can be guaranteed from the beginning of the process and the reaching phase is eliminated. Numerous research results of this technique can be found over the last few years. For example, in [19], a robust ISMC design for the uncertain stochastic system with state delay was studied by means of the feasibility of LMIs. In [20], combining ISMC with robust model predictive control, a control scheme was proposed for nonlinear constrained continuous-time uncertain systems. In particular, some research results based on ISMC have been developed for FTC systems. In [21], the active FTC issues were studied from the ISMC viewpoint to accommodate actuator faults whenever the fault detection and diagnosis information was available. With consideration of managing actuator redundancy, an integral-type sliding mode fault-tolerant controller incorporated with control allocation was developed to handle the total failure of certain actuators for a class of over-actuated linear systems [22].

Inspired by the above research, this paper investigates ISMC-based FTC strategies for rigid spacecraft attitude stabilization in the presence of external disturbances and two kinds of actuator faults. Specifically, for the zero-disturbance fault-free attitude control system, a simple saturated proportional-derivative (PD) control law which is regarded as nominal controller, is proposed to asymptotically stabilize the attitude motion, where a scalar sharpness function is contained to improve dynamic performance. Then, bounded external disturbances and two kinds of actuator faults, including both

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partial loss of control effectiveness fault and additive fault, are taken into account for attitude dynamics. Assuming that there exists a priori knowledge of the bounds of disturbances and actuator faults, a basic ISMC-based fault-tolerant controller is proposed to maintain close to the nominal closed-loop performance from the beginning of the process. Moreover, by means of adaptive mechanism, an adaptive ISMC-based FTC scheme is designed such that the resultant closed-loop system is capable of tolerating potential actuator faults without requiring any information on the boundaries of disturbances and faults except for their existence. The main contributions of this paper are stated as follows:

1) Actuator faults and external disturbances can be completely compensated from the initial time instant when basic FTC scheme is used.
2) The proposed adaptive FTC scheme has a simple structure and only one parameter is required to be adapted on-line, thus simplifying the design process and reducing the online computation load significantly.
3) In conventional adaptive SMC, the sliding manifold deviation which is mainly caused by initial state error, is the main reason for the increase in the estimated switching gain [21], [23]. Since the proposed adaptive controller is within the ISMC framework, one can see that the initial sliding manifold deviation can be removed during the switching gain adaptation process, and thus the obtained control magnitude is less demanding than that of classical adaptive SMC designs.

The remainder of this paper is organized as follows. In Section II, spacecraft attitude dynamics and actuator fault model are described, and a nominal controller based on simple saturated PD control is proposed. In Section III, basic ISMC-based FTC scheme and adaptive ISMC-based FTC scheme are presented, respectively, with and without requiring the knowledge of the boundary of the external disturbances and faults. The simulation results are given in Section IV, followed by conclusions in Section V.

II. Preliminaries

A. Spacecraft Attitude Dynamics and Kinematics

The kinematics and dynamics for the attitude motion of a rigid spacecraft can be expressed by the following equations [24]:

$$J\dot{\omega} = -\omega^{\times} J\omega + \tau + d \tag{1}$$

$$\dot{q}_v = \frac{1}{2}({q_v}^{\times} + q_0 I_3)\omega \tag{2}$$

$$\dot{q}_0 = -\frac{1}{2}{q_v}^T \omega \tag{3}$$

where $J = J^T$ denotes the positive definite inertia matrix of the spacecraft, $\omega \in \mathbb{R}^3$ is the inertial angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, $q = [q_0, q_1, q_2, q_3]^T = [q_0, q_v^T]^T \in \mathbb{R} \times \mathbb{R}^3$ denotes the unit quaternion describing the attitude orientation of the body frame $\mathcal{B}$ with respect to inertial frame $\mathcal{I}$ and satisfies the constraint $q_0^2 + q_v^T q_v = 1$, $I_3 \in \mathbb{R}^{3\times 3}$ denotes the identity matrix, $\tau \in \mathbb{R}^3$ and $d \in \mathbb{R}^3$ denote the control torque and the external disturbances respectively. The notation $a^{\times} \in \mathbb{R}^{3\times 3}$ for a vector $a = [a_1, a_2, a_3]^T$ is used to represent the skew-symmetric matrix

$$a^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{4}$$

Suppose that the internal dynamics in actuators is negligible. An actuator with its output equal to its input is regarded as fault free. The actuator faults of interest are modeled as follows:

$$\tau = [I_3 - E(t)]\, u + \bar{u} \tag{5}$$

where $E(t) = diag\{e_1(t), e_2(t), e_3(t)\} \in \mathbb{R}^{3\times 3}$ denotes the effectiveness loss factor matrix for the spacecraft actuators with $0 \le e_i(t) < 1 (i = 1, 2, 3)$. Note that the case $e_i(t) = 0$ indicates that the ith actuator works normally, and $0 < e_i(t) < 1$ implies that the ith actuator partially loses its effectiveness, but still not totally fail. $\overline{u} \in \mathbb{R}^3$ represents the bounded time-varying additive actuator fault. Such an actuator fault formulation can be found in many previous results, such as [7], [25], and [26]. Hence, the nonlinear attitude dynamics model incorporating actuator faults defined in (5) can be rewritten as the following form:

$$J\dot{\omega} = -\omega^{\times} J\omega + [I_3 - E(t)]\, u + \bar{u} + d. \tag{6}$$

To facilitate the controller development, the actuator faults and the external disturbances are assumed to satisfy the following Assumptions.

Assumption 1: The external disturbance d is bounded such that $\|d\| \le d_{max}$, where d_{max} is a positive constant and $\|\cdot\|$ denotes the Euclidean norm.

Assumption 2: $0 \le max\{e_1, e_2, e_3\} \le e_m < 1$, where e_m is a positive constant.

Assumption 3: The actuator additive fault may be time-varying and unknown, but it is always bounded such that $\|\bar{u}\| \le f_m$, where f_m is a positive constant.

B. Nominal Controller

For the zero-disturbance fault-free attitude control system, the following nominal controller is developed and will be used in Sec. III for the proposed fault-tolerant controller design.

Lemma 1: Consider the normal attitude dynamics without any actuator faults and external disturbances

$$J\dot{\omega} = -\omega^{\times} J\omega + \tau \tag{7}$$

if the nominal controller law is designed as the following simple saturated PD controller:

$$u_{nom} = -k_p q_v - k_d Tanh\left(\frac{\omega}{p^2}\right) \tag{8}$$

where k_p and k_d are positive constants, $Tanh\left(\frac{\omega}{p^2}\right) = \left[tanh\left(\frac{\omega_1}{p^2}\right),\, tanh\left(\frac{\omega_2}{p^2}\right),\, tanh\left(\frac{\omega_3}{p^2}\right)\right]^T$, p^2 is a nonzero scalar sharpness function satisfying $0 < p_{min}^2 \le p^2 \in \ell_\infty$ and $\dot{p} \in \ell_\infty$. Then, the zero-disturbance closed-loop system is asymptotically stable, i.e., $q_v \to 0$, $\omega \to 0$ as $t \to \infty$.

Proof. Consider the following positive-definite radially unbounded candidate Lyapunov function:

$$V = k_p(1-q_0)^2 + k_p q_v^T q_v + \frac{1}{2}\omega^T J\omega. \tag{9}$$

The first derivative along the motion of (2), (3), and (7) is given by

$$\begin{aligned}\dot{V} &= -2k_p\dot{q}_0 + \omega^T J\dot{\omega} = k_p q_v^T\omega + \omega^T(-\omega^\times J\omega + u_{nom}) \\ &= k_p q_v^T\omega - k_p\omega^T q_v - k_d\omega^T Tanh(\omega/p^2) \\ &= -k_d\omega^T Tanh(\omega/p^2). \end{aligned} \tag{10}$$

By using the fact that $x\tanh(x/p^2) \geq 0$ for all x, $\dot{V} \leq 0$ can be obtained. Therefore, one can show that q_0, q_v, and ω are globally bounded. In view of the constraints on p^2, it is clear that $\ddot{V}$ is bounded. Hence, according to the Barbalat's Lemma [27], one can conclude that $\lim_{t\to\infty}\omega(t) = 0$. At this point, it remains to be shown that $\lim_{t\to\infty} q_v(t) = 0$. From (7) and (8), it can be derived that

$$\begin{aligned} J\ddot{\omega} &= -\dot{\omega}^\times J\omega - \omega^\times J\dot{\omega} + \dot{u}_{nom} \\ &= -\dot{\omega}^\times J\omega - \omega^\times J\dot{\omega} - k_p\dot{q}_v - k_d\frac{d}{dt}Tanh(\frac{\omega}{p^2}). \end{aligned} \tag{11}$$

Now, consider the last term in (11)

$$\frac{d}{dt}tanh(\frac{\omega_i}{p^2}) = sech^2\left(\frac{\omega_i}{p^2}\right)\frac{\dot{\omega}_i p - 2\omega_i\dot{p}}{p^3} \tag{12}$$

where ω_i is the ith element of ω $(i = 1, 2, 3)$. Since ω, $\dot{\omega}$, $\dot{q}_v$, and $\dot{p}$ are bounded and $0 < p^2_{min} \leq p^2$, from (11) and (12), one can show that $\ddot{\omega}$ is bounded. Consequently, by virtue of the Barbalat's Lemma again, together with convergence of ω to the origin and uniform continuity of $\dot{\omega}$, it leads to $\lim_{t\to\infty}\dot{\omega}(t) = 0$. As t goes to infinity, from (7), we have $u_{nom} = 0$. Therefore, from (8), it is clear that $\lim_{t\to\infty}\left[k_p q_v + k_d Tanh(\frac{\omega}{p^2})\right] = 0$, which implies that $\lim_{t\to\infty} q_v(t) = 0$ since $\lim_{t\to\infty}\omega(t) = 0$. Thus, the results as stated in Lemma 1 is established. □

Remark 1: Using the property of the unit quaternion and the standard hyperbolic tangent function, the u_{nom} in (8) can be upper bounded as

$$|u_{nomi}| \leq |k_p q_{vi}| + \left|k_d Tanh(\omega_i/p^2)\right| \leq k_p + k_d \tag{13}$$

where u_{nomi}, q_{vi}, and ω_i represent the ith element of u_{nom}, q_v, and ω, respectively. Hence, a natural saturation, in terms of the nominal control gains, is achieved and the designer can set the limits of the nominal control effort through the gains k_p and k_d.

Remark 2: In comparison with the similar results presented in [28], where Rodrigues parameters and modified Rodrigues parameters are used for the attitude representation, a nonzero sharpness function p^2 is introduced in the proposed nominal controller. It should be noted that p^2 appears in the denominator of the argument of the hyperbolic tangent function, so that its value determines how strongly the control varies with the signal ω. Adjusting p^2 changes the rate of change of nominal control torque, and thus, better dynamic performance could be obtained [29].

III. Fault-Tolerant Attitude Controller Design

In this section, based on the existing results on the integral-type sliding mode manifold, the ISMC-based FTC schemes are developed to solve the attitude stabilization problem for rigid spacecraft under actuator faults and external disturbances.

A. Integral-Type Sliding Manifold Design

Adopting the integral-type sliding mode manifold design procedure, the sliding manifold is introduced as follows:

$$s = D\Big\{\omega(t) - \omega(t_0) - \int_{t_0}^{t} J^{-1}\left[-\omega(\sigma)^\times J\omega(\sigma) + u_{nom}(q,\omega)\right]d\sigma\Big\} \tag{14}$$

where $D \in \mathbb{R}^{3\times 3}$ is a constant matrix. The matrix D is chosen such that DJ^{-1} is invertible. Notice that, at $t = t_0$, the sliding manifold satisfies $s(\omega(t_0), t_0) = 0$, and hence the reaching phase is eliminated [16]–[18].

In order to analyze the sliding dynamics (motion equations on the sliding manifold) in the presence of actuator faults, the equivalent control method [14] is used. Taking the derivative of the sliding manifold in (14) with respect to time yields

$$\dot{s} = D\left[\dot{\omega} - J^{-1}(-\omega^\times J\omega + u_{nom})\right]. \tag{15}$$

Substituting the attitude dynamics model with actuator faults model defined in (6) into (15), it follows that

$$\dot{s} = DJ^{-1}\left[(I_3 - E(t))u + \bar{u} - u_{nom} + d\right]. \tag{16}$$

To derive the equivalent control, the algebraic equation $\dot{s} = 0$ should be solved, which gives

$$u_{eq} = [I_3 - E(t)]^{-1}(u_{nom} - \bar{u} - d) \tag{17}$$

where DJ^{-1} is invertible. Thus, the sliding dynamics can be obtained by substituting (17) into (6) as

$$J\dot{\omega}_h = -\omega_h^\times J\omega_h + u_{nom} \tag{18}$$

where the subscript h denotes the state vector in the sliding mode. Furthermore, according to the Lemma 1, it can be concluded that the sliding dynamics in (18) is asymptotically stable.

Remark 3: Based on the aforementioned analysis, it is found that actuator faults and external disturbances can be completely rejected when the dynamics of spacecraft remains on the sliding manifold. That is, the closed-loop dynamics of the faulty system in the sliding mode is identical to that of the zero-disturbance healthy system controlled by the nominal controller. Hence, the designer has much more freedom in designing a suitable nominal controller according to the system requirements for the nominal healthy system in the absence of external disturbances.

B. Basic Integral-Type Sliding Mode FTC Design

Now, a sliding mode control law must be synthesized such that the reachability of the specified integral-type sliding manifold is ensured under partial loss of actuator effectiveness fault and additive actuator fault.

The proposed basic ISMC-based FTC scheme has a form given by

$$u = u_{nom} + u_N \tag{19}$$

where the nominal control u_{nom} is the same as (8) which determines the behavior of the nominal system restricted to the sliding manifold, and the second term u_N is a discontinuous control component that compensates for the possible actuator fault effects on the system and drives the system trajectories toward the sliding manifold.

In this section, the upper bounds of actuator effectiveness fault and additive fault in Assumption 1 and 2 are assumed to be known to the designer. Thus, the discontinuous control u_N is selected as

$$u_N = \begin{cases} -\rho(t)\frac{(DJ^{-1})^T s}{\|(DJ^{-1})^T s\|}, & if\ s \neq 0 \\ 0, & otherwise \end{cases} \tag{20}$$

with the switching gain function

$$\rho(t) = \frac{\sqrt{3}e_m\|u_{nom}\|_\infty + f_m + d_{max} + \varepsilon}{1 - e_m} > 0 \tag{21}$$

where ε is a bounded positive constant.

Theorem 1: Consider the attitude control systems described by (1-3) in the presence of partial loss of actuator effectiveness fault and additive actuator fault. Suppose that Assumptions 1-3 are valid with known bounds on actuator faults and external disturbances. Then the reachability of the sliding manifold $s = 0$ can be maintained by employing the controller in (19) with u_{nom} and u_N given in (8) and (20), respectively.

Proof. To analyze the reachability, consider the candidate Lyapunov function

$$V = \frac{1}{2}s^T s. \tag{22}$$

Taking the time derivative of the Lyapunov function for $s \neq 0$ with substitution of FTC law in (19), yields

$$\begin{aligned} \dot{V} &= s^T DJ^{-1}\Big[-(I_3 - E(t))\rho(t)\frac{(DJ^{-1})^T s}{\|(DJ^{-1})^T s\|} \\ &\quad - E(t)u_{nom} + \bar{u} + d\Big] \\ &\leq -\big[(1-e_m)\rho(t) - \sqrt{3}e_m\|u_{nom}\|_\infty \\ &\quad - f_m - d_{max}\big]\big\|(DJ^{-1})^T s\big\| \end{aligned} \tag{23}$$

where the inequality $x^T y \leq \sqrt{3}\|x\|\|y\|_\infty$ with $x \in \mathbb{R}^3$ and $y \in \mathbb{R}^3$ has been used. Furthermore, since DJ^{-1} is nonsingular, substituting for $\rho(t)$ from (21) into (23) gives

$$\dot{V} \leq -\varepsilon\big\|(DJ^{-1})^T s\big\| < 0, \qquad for\ s \neq 0. \tag{24}$$

This implies that the sliding motion can be maintained despite partial loss of actuator effectiveness fault and additive actuator fault as well as external disturbances. This completes the proof. □

C. Integral-Type Sliding Mode FTC Design with Adaptive Estimation

It is seen that the upper bounds d_{max}, e_m, and f_m of external disturbances and actuator faults are required to synthesize the basic FTC law in (19). However, from a practical point of view, the exact knowledge of these bounds is not easily obtained due to the uncertain and unexpected characteristics of disturbances and faults. Therefore, an adaptive integral sliding mode fault-tolerant controller is further developed by incorporating parameter adaptive mechanism to relax the requirement of a priori knowledge of these bounds.

Notice that the switching gain function $\rho(t)$ in (21) is upper bounded with the property that

$$\rho(t) \leq \rho_m = \frac{\sqrt{3}e_m(k_p + k_d) + f_m + d_{max} + \varepsilon}{1 - e_m} \tag{25}$$

where ρ_m is a positive scalar representing the upper bound of the switching gain function $\rho(t)$.

Remark 4: Recalling the switching gain $\rho(t)$ defined in (21), from Assumption 3 and Remark 2, it is clear that $\rho(t) \leq \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}+\varepsilon}{1-e_m}$. Since k_p and k_d are bounded positive constant design parameters, and e_m, f_m, d_{max} and ε are bounded positive constants, the positive scalar ρ_m always exists. Therefore, the upper bounded property of the switching gain $\rho(t)$ can be obtained.

Next, we shall introduce an adaptive scheme which is capable of performing an estimation of the upper bound ρ_m and design a suitable adaptive fault-tolerant controller using this estimated upper bound. Based on the aforementioned analysis, the basic fault-tolerant control law in (19) is modified as

$$u = u_{nom} + u_{aN} \tag{26}$$

where the nominal control u_{nom} is the same as (8). Due to the discontinuity of u_N in (19), the basic ISMC-based FTC is discontinuous across the sliding manifold $s = 0$, thus leading to control chattering. In order to alleviate undesirable chattering, the discontinuous control in (20) is smoothed by

$$u_{aN} = \begin{cases} -\hat{\rho}_m\frac{(DJ^{-1})^T s}{\|(DJ^{-1})^T s\|}, & if\ \hat{\rho}_m\big\|(DJ^{-1})^T s\big\| \geq \xi \\ -\hat{\rho}_m^2\frac{(DJ^{-1})^T s}{\xi}, & if\ \hat{\rho}_m\big\|(DJ^{-1})^T s\big\| < \xi \end{cases} \tag{27}$$

where ξ is a small positive scalar, and $\hat{\rho}_m \geq 0$ is obtained by the following adaptive law:

$$\dot{\hat{\rho}}_m = \beta(\big\|(DJ^{-1})^T s\big\| - \mu\hat{\rho}_m), \text{ with } \hat{\rho}_m(0) \geq 0 \tag{28}$$

where β and μ are positive scalars, and the second term $-\mu\hat{\rho}_m$ is used to establish robustness with respect to disturbances and unmodeled dynamics [30] and prevent the increase of the adaptive gain [31], [32].

Lemma 2: Given the switching gain update law in (28), the gain $\hat{\rho}_m$ has an upper bound, i.e., there always exists a positive scalar $\bar{\rho}_m$ such that $\hat{\rho}_m \leq \bar{\rho}_m$ and $\rho_m \leq \bar{\rho}_m$ for all $t > 0$.

Proof. See the Appendix. □

Theorem 2: Consider the attitude control systems described by (1-3) in the presence of partial loss of actuator effectiveness fault and additive actuator fault. Suppose that Assumptions 1-3 are valid with unknown upper bounds on the actuator faults (only the boundness property is known). Then the trajectory of the closed-loop system can be driven into a neighborhood of the sliding manifold $s = 0$ in finite time by employing the adaptive controller in (26) and the parameter adaptive law in (28).

Proof. Consider the following candidate Lyapunov function:

$$V = \frac{1}{2}s^T s + \frac{1-e_m}{2\beta}(\hat{\rho}_m - \bar{\rho}_m)^2 \tag{29}$$

where $\bar{\rho}_m$ is defined in Assumption 4 representing the upper bound of $\hat{\rho}_m$.

Case I: If $\hat{\rho}_m\left\|(DJ^{-1})^T s\right\| \geq \xi$, the time derivative of V along with (26)-(28) results in

$$\begin{aligned}\dot{V} =& s^T\dot{s} + \frac{1-e_m}{\beta}(\hat{\rho}_m - \bar{\rho}_m)\dot{\hat{\rho}}_m \\ \leq& -(1-e_m)\left[\bar{\rho}_m - \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}}{1-e_m}\right] \\ &\times \left\|(DJ^{-1})^T s\right\| - \mu(1-e_m)(\hat{\rho}_m - \bar{\rho}_m)\hat{\rho}_m. \end{aligned} \tag{30}$$

where the inequality $x^T y \leq \sqrt{3}\|x\|\|y\|_\infty$ with $x \in \mathbb{R}^3$ and $y \in \mathbb{R}^3$ has been used. In view of Lemma 2, it is clear that $\bar{\rho}_m - \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}}{1-e_m} \geq \frac{\varepsilon}{1-e_m}$ and $|\hat{\rho}_m - \bar{\rho}_m| \leq \bar{\rho}_m$. Now, it is readily obtained from (30) that

$$\begin{aligned}\dot{V} \leq& -\mu(1-e_m)|\hat{\rho}_m - \bar{\rho}_m| - \varepsilon\left\|(DJ^{-1})^T s\right\| \\ &+ \mu(1-e_m)(|\hat{\rho}_m - \bar{\rho}_m| + \frac{1}{4}\bar{\rho}_m^2) \\ =& -\delta_1|\hat{\rho}_m - \bar{\rho}_m| - \delta_2\|s\| + \eta_1,\end{aligned}$$

where δ_1, δ_2, and η_0 are positive scalars defined by $\delta_1 = \mu(1-e_m)$, $\delta_2 = \varepsilon\left\|DJ^{-1}\right\|$, and $\eta_1 = \mu(1-e_m)(\bar{\rho}_m + \frac{1}{4}\bar{\rho}_m^2)$, respectively. Thus, it follows that

$$\begin{aligned}\dot{V} \leq& -\sqrt{\frac{2\beta\delta_1^2}{1-e_m}}\sqrt{\frac{1-e_m}{2\beta}(\hat{\rho}_m - \bar{\rho}_m)^2} - \sqrt{2}\delta_2\sqrt{\frac{1}{2}s^T s} + \eta_1 \\ \leq& -\min\left\{\sqrt{\frac{2\beta\delta_1^2}{1-e_m}}, \sqrt{2}\delta_2\right\} \\ &\times\left(\sqrt{\frac{1-e_m}{2\beta}(\hat{\rho}_m - \bar{\rho}_m)^2} + \sqrt{\frac{1}{2}s^T s}\right) + \eta_1 \\ \leq& -\delta V^{\frac{1}{2}} + \eta_1,\end{aligned} \tag{31}$$

where $\delta = \sqrt{2}\min\left\{\sqrt{\frac{\beta\delta_1^2}{1-e_m}}, \delta_2\right\}$.

Case II: If $\hat{\rho}_m\left\|(DJ^{-1})^T s\right\| < \xi$, the time derivative of the Lyapunov candidate function defined in (29) is

$$\begin{aligned}\dot{V} =& -\frac{1-e_m}{\xi}\hat{\rho}_m^2\|(DJ^{-1})^T s\|^2 + (1-e_m)\hat{\rho}_m\|(DJ^{-1})^T s\| \\ &-(1-e_m)\left[\hat{\rho}_m - \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}}{1-e_m}\right] \\ &\times\left\|(DJ^{-1})^T s\right\| + (1-e_m)(\hat{\rho}_m - \bar{\rho}_m)\left\|(DJ^{-1})^T s\right\| \\ &-\mu(1-e_m)(\hat{\rho}_m^2 - \hat{\rho}_m\bar{\rho}_m).\end{aligned} \tag{32}$$

Since $\hat{\rho}_m\left\|(DJ^{-1})^T s\right\| < \xi$, it is easy to prove that the term $-\frac{1-e_m}{\xi}\hat{\rho}_m^2\|(DJ^{-1})^T s\|^2 + (1-e_m)\hat{\rho}_m\|(DJ^{-1})^T s\|$ reaches its maximum value of $\frac{(1-e_m)\xi}{4}$ when $\hat{\rho}_m\left\|(DJ^{-1})^T s\right\| = \frac{\xi}{2}$. Then, with the help of Lemma 2 and follow the same lines as in *case I*, it can be shown from (32) that

$$\dot{V} \leq -\delta V^{\frac{1}{2}} + \eta_2. \tag{33}$$

where δ_1 and δ_2 have the same definitions as in case I, and η_2 is a positive scalar defined by $\eta_2 = \frac{1-e_m}{4}(\mu\bar{\rho}_m^2 + 4\mu\bar{\rho}_m + \xi)$.

Therefore, combining above two cases together, for any $\hat{\rho}_m\left\|(DJ^{-1})^T s\right\|$, one has

$$\dot{V} \leq -\delta V^{\frac{1}{2}} + \eta, \tag{34}$$

where $\eta = \max\{\eta_1, \eta_2\} = \frac{1-e_m}{4}(\mu\bar{\rho}_m^2 + 4\mu\bar{\rho}_m + \xi)$. Thus, the trajectory of this system is practically finite-time stable [33]. Furthermore, the decrease of V can in finite time drive the trajectories of the closed-loop system into $V^{\frac{1}{2}} \leq \frac{\eta}{(1-\theta_0)\delta}$, where θ_0 is a positive scalar satisfying $0 < \theta_0 < 1$. As a result, the trajectories of the closed-loop system is bounded in a small set containing the origin in finite time as

$$\begin{aligned}\|s\| \leq& \frac{\sqrt{2}\eta}{(1-\theta_0)\delta} \\ =& \frac{(1-e_m)(\mu\bar{\rho}_m^2 + 4\mu\bar{\rho}_m + \xi)}{4(1-\theta_0)\cdot\min\left\{\mu\sqrt{\beta(1-e_m)}, \varepsilon\|DJ^{-1}\|\right\}}.\end{aligned} \tag{35}$$

This completes the proof. □

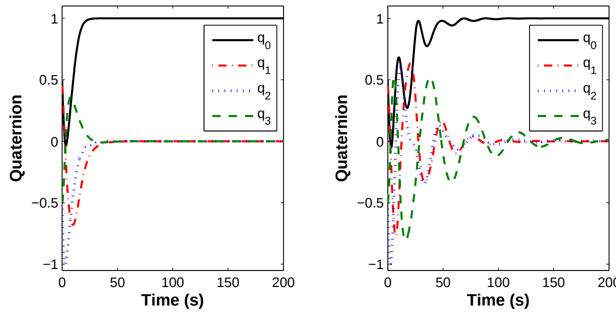
Remark 5: From inequality (35), it can be seen that the smaller the boundary thickness ξ, the smaller the desired s is obtained. Furthermore, the ultimate convergence set for s can be small enough around the origin by choosing large enough parameters β and D.

Remark 6: If a sufficiently large value for $\hat{\rho}_m(0)$ can be obtained to compensate the potential disturbance and fault in the systems, the sliding function will stay in the small convergence set even after a fault occurs. However, because the magnitude of potential actuator fault in the system is always unknown to the designer, a proper initial value of the adaptive gain $\hat{\rho}_m(0)$ is difficult to select. In this case, although the sliding function may move out of the small convergence set after a fault occurs, it will be attracted back to the small convergence set in finite time.

IV. Simulation Results

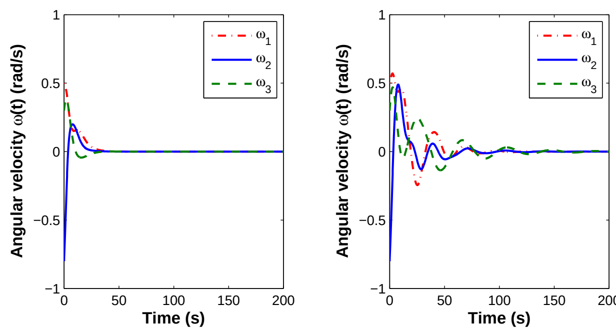
Simulation results are presented in this section to illustrate the effectiveness of the proposed ISMC-based FTC schemes. Similar to [34], the inertia matrix of a rigid spacecraft is given as $J = diag\{10, 15, 20\}$ kg · m^2. The external disturbances are of the form $d = 0.1[\sin(t/10), \cos(t/15), \sin(t/20)]^T + [0.1,\ 0.1,\ 0.1]^T$ Nm. The initial attitude and angular velocity are set as $q(0) = [0.5, 0.5, -0.5, -0.5]^T$ and $\omega(0) = [0.5, -0.8, 0.3]^T$ rad/s, respectively.

In the context of simulation, a severe fault scenario is considered. At $t = 10$ s, each actuator suffers from a partial loss of effectiveness fault, while at $t = 50$ s, these actuators also undergo an additive time-varying fault that enters



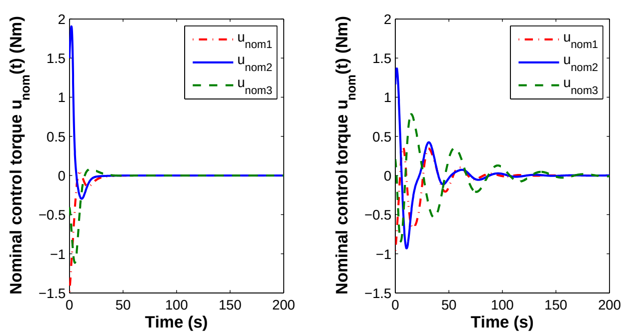
(a) $p^2 = 0.2$ (b) $p^2 = 1$

Fig. 1. Time response of quaternion with different sharpness parameters in the absence of actuator faults and external disturbances.



(a) $p^2 = 0.2$ (b) $p^2 = 1$

Fig. 2. Time response of angular velocity with different sharpness parameters in the absence of actuator faults and external disturbances.



(a) $p^2 = 0.2$ (b) $p^2 = 1$

Fig. 3. Time response of control input with different sharpness parameters in the absence of actuator faults and external disturbances.

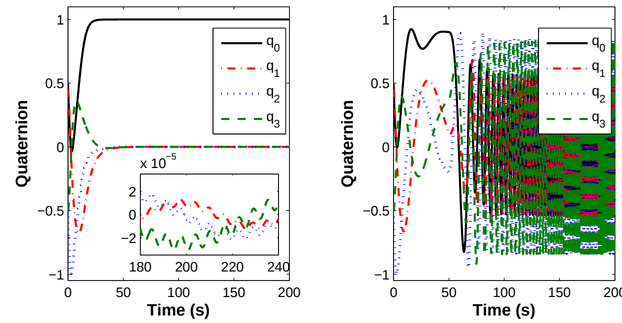
the spacecraft dynamics in an additive way. The details are described as follows:

$$e_i(t) = \begin{cases} 0, & if\ t < 10 \\ 0.5, & if\ t \geq 10 \end{cases}$$

$$\bar{u}_i(t) = \begin{cases} 0, & if\ t < 50 \\ 0.95 + 0.05sin(t), & if\ t \geq 50. \end{cases}$$

A. Nominal Saturated PD Controller

In order to demonstrate the effectiveness of the nominal saturated PD controller, the nominal situation is simulated in which all actuators are healthy and there is no external disturbance. The gains for the nominal controller are chosen as $k_p = 1$ and $k_d = 1$. For convenience, the sharpness function p^2 is set as a constant. To analyze the effect of changes in p^2, the nominal controller is applied twice with different sharpness parameters (constant p^2) in the simulation.



(a) Basic FTC scheme (b) Saturated PD scheme

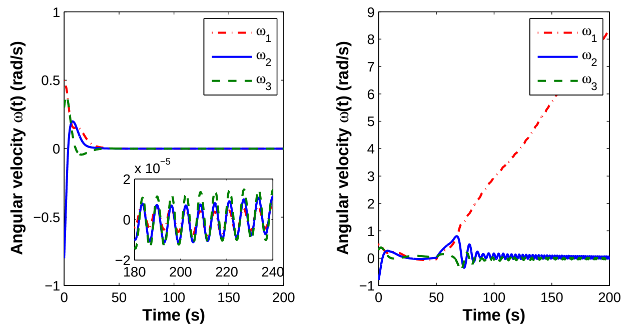
Fig. 4. Time response of quaternion under different control schemes in the presence of actuator faults and external disturbances.

The response of attitude quaternion and angular velocity are shown in Figs. 1 and 2, from which it is clear that asymptotic attitude stabilization of the spacecraft is achieved by applying the proposed nominal controller. Fig. 3 depicts the time history of the nominal control effort u_{nom} given by (8), which shows that the applied nominal control torque remains bounded ($|u_{nomi}| \leq k_p + k_d = 2$ Nm for all time). In addition, referring to Figs. 1-3, it is shown that the sharpness parameter affects the transient performance greatly and, more specifically, decreasing p^2 can improve the closed-loop convergence rate and reduce the overshoot.

B. Basic Fault-Tolerant Controller

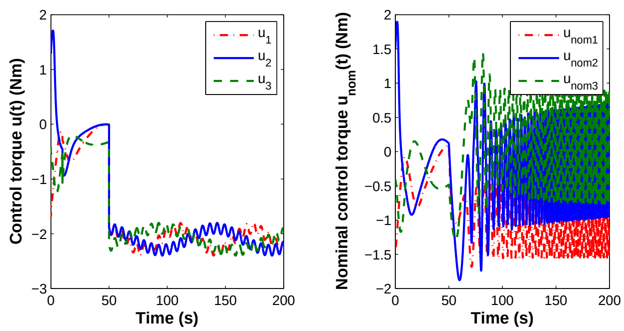
For the basic controller proposed in (19), the control parameters are chosen as $k_p = 1$, $k_d = 1$, $p^2 = 0.2$, and $\varepsilon = 1$, while the parameter of the integral-type sliding mode manifold is $D = 2I_3$. In order to eliminate control chattering, the discontinuous control action in (20) is smoothed by using the boundary modification [35], and the boundary layer width is chosen as 0.0001. For the purpose of comparison, the nominal saturated PD controller, which does not have a mechanism to accommodate the actuator fault, is also applied to the spacecraft attitude control system in the presence of actuator faults with $k_p = 1$, $k_d = 1$, and $p^2 = 0.2$.

Figs. 4 and 5 show the attitude quaternion and angular velocity trajectory. It can be observed that the basic ISMC-based fault-tolerant controller obtains a good performance in the attitude stabilization even in the presence of partial loss of actuator effectiveness fault and additive actuator fault. As compared with the basic FTC case, the attitude control systems can not be stabilized by the nominal saturated PD controller when actuator faults are introduced in the system, especially after the occurrence of additive actuator fault in 50s. From Fig. 6, it is shown that the basic FTC scheme in (19) manages to produce a control torque to accommodate the effect of actuator faults. On the other hand, for the nominal controller case, it is observed that the effect of actuator faults can not be compensated by the nominal saturated PD controller, which may result in severe control chattering.



(a) Basic FTC scheme (b) Saturated PD scheme

Fig. 5. Time response of angular velocity under different control schemes in the presence of actuator faults and external disturbances.



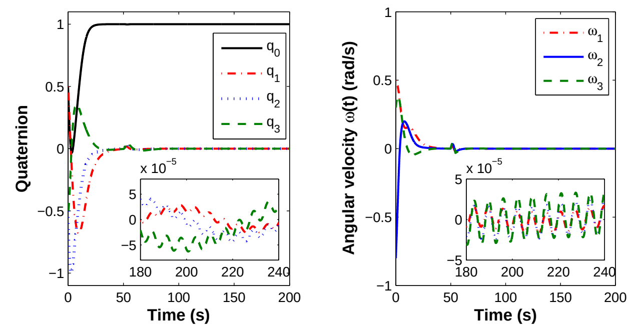
(a) Basic FTC scheme (b) Saturated PD scheme

Fig. 6. Time response of control input under different control schemes in the presence of actuator faults and external disturbances.

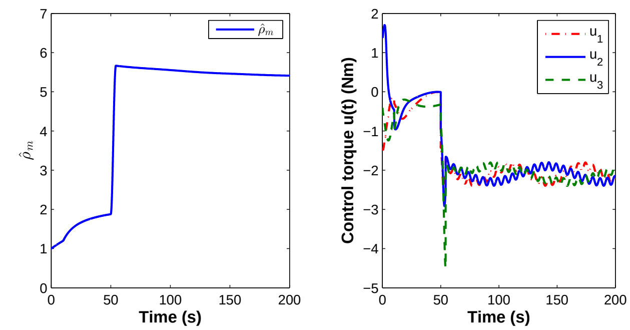
C. Adaptive Fault-Tolerant Controller

To illustrate the effectiveness of the proposed adaptive fault-tolerant controller defined in (26), the attitude stabilization problem in the absence of a priori knowledge of upper bounds on disturbances and the actuator faults is simulated in this section. In the simulation, the same gains of $k_p = 1$, $k_d = 1$ and $p^2 = 0.2$ as in the case of basic controller are used for the nominal saturated PD controller. The gain of integral-type sliding mode manifold is chosen as $D = 5I_3$, and the boundary layer width is chosen as $\xi = 0.01$. The design parameters of the adaptive law are chosen as $\beta = 10$ and $\mu = 0.00025$ with initial value $\hat{\rho}_m(0) = 1$.

From Figs. 7a and 7b, it is shown that the overall attitude quaternion and angular velocity trajectories are stabilized within 110 s, and acceptable performance is also achieved. Figs. 7c and 7d depict the time histories of the estimate of the switching gain $\hat{\rho}_m(t)$ and the control torque u given by (26). Referring to Figs. 7a-7d, it is clear that the proposed adaptive FTC scheme is capable of accommodating actuator faults even under the condition that the upper bounds of the actuator faults are unknown. However, compared with the basic FTC scheme, it is observed that the convergence of the system states takes a much longer time when adaptive FTC scheme is used. This is due to the fact that the estimated switching gain requires a period of time to become a value large enough to compensate for the efforts of the actuator fault once a fault happens.



(a) Quaternion (b) Angular velocity



(c) Adaptive parameters $\hat{\rho}_m(t)$ (d) Control input

Fig. 7. Simulation results under adaptive FTC scheme in the presence of actuator faults and external disturbances.

V. Conclusion

In this paper, two ISMC-based FTC schemes for the attitude stabilization problem of a rigid spacecraft subject to two kinds of actuator faults and external disturbances have been proposed. In zero-disturbance fault-free case, a simple saturated PD control law is proposed to achieve asymptotic attitude stabilization. When actuator faults occur in the system, the basic FTC scheme completely compensates the effect of actuator faults from the beginning of the process. In the event that the bounds on the actuator faults and disturbances are unknown, an adaptive fault-tolerant controller is proposed to ensure that the trajectory of the system is practical finite-time stable with a reasonable switching adaptive gain. Numerical simulations are performed to verify the fault-tolerant capability of the proposed FTC laws in the presence of two kinds of actuator faults and external disturbances. In future work, control input saturation and actuator redundancy should be investigated to reduce the power consumption and improve system reliability, respectively.

Appendix

Proof of Lemma 2. Consider a Lyapunov function $V_s = \frac{1}{2}s^T s$. Differentiating V_s with respect to time and substituting the closed-loop equations into it, we have

$$\dot{V}_s \leq -(1-e_m)\left(\hat{\rho}_m(t)-\rho_m\right)\left\|(DJ^{-1})^T s\right\| \qquad (36)$$

where $\rho_m = \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}+\varepsilon}{1-e_m}$ defined in equation (25) is a finite constant. To show $\hat{\rho}_m$ is upper bounded, the following four cases are considered here.

Case 1: If $\hat{\rho}_m(t) > \rho_m$ and $\|(DJ^{-1})^T s\| \geq \mu\rho_m$, it follows that $\dot{V}_s(t) < 0$ and $\dot{\hat{\rho}}_m(t) \geq 0$. Since $\dot{V}_s(t) \leq 0$, the sliding function s will decrease and it can be obtained that $V_s(t) \leq V_s(0)$, which implies that $\|s\| \leq \sqrt{2V_s(0)}$. Therefore, although $\dot{\hat{\rho}}_m \geq 0$, the adaptive gain $\hat{\rho}_m$ is upper bounded in view of the updating law $\dot{\hat{\rho}}_m = \beta(\|(DJ^{-1})^T s\| - \mu\hat{\rho}_m)$.

Case 2: If $\hat{\rho}_m(t) > \rho_m$ and $\|(DJ^{-1})^T s\| < \mu\rho_m$, it is clear that $\dot{V}_s(t) < 0$ and $\dot{\hat{\rho}}_m(t) \leq 0$. As a result, both the Lyapunov function $V_s(t)$ and the adaptive gain $\hat{\rho}_m(t)$ are upper bounded by their initial values $V_s(0)$ and $\hat{\rho}_m(0)$, respectively, i.e., $V_s(t) \leq V_s(0)$ and $\hat{\rho}_m(t) \leq \hat{\rho}_m(0)$.

Case 3: If $\hat{\rho}_m(t) \leq \rho_m$ and $\|(DJ^{-1})^T s\| < \mu\rho_m$, it follows that $\dot{V}_s(t) > 0$ and $\dot{\hat{\rho}}_m(t) \leq 0$. In this case, it is obvious that $\hat{\rho}_m$ is upper bounded by the constant ρ_m. Moreover, although $\dot{V}_s > 0$, we shall further prove that V_s is bounded. Since the sliding function s is under the condition that $\|(DJ^{-1})^T s\| \leq \mu\rho_m$ is satisfied, it could be shown that $\|s\| \leq \frac{\mu\rho_m}{\|(DJ^{-1})^T\|}$. Hence, the Lyapunov function $V_s(t)$ is upper bounded by a constant $V_s^* = \frac{1}{2}\left(\frac{\mu\rho_m}{\|(DJ^{-1})^T\|}\right)^2$ in this case.

Case 4: If $\hat{\rho}_m(t) \leq \rho_m$ and $\|(DJ^{-1})^T s\| \geq \mu\rho_m$, it is clear that $\dot{V}_s(t) > 0$ and $\dot{\hat{\rho}}_m \geq 0$. It is also obvious that $\hat{\rho}_m$ is upper bounded by the constant ρ_m in this case. Since s is a large value such that $\|(DJ^{-1})^T s\| > \mu\rho_m$, the adaptive law could be approximated as $\dot{\hat{\rho}}_m(t) = \beta(\|(DJ^{-1})^T s\|$. Comparing the expressions of $\dot{V}_s(t)$ and $\dot{\hat{\rho}}_m(t)$, it is noted that $\dot{V}_s(t) \leq k_m\dot{\hat{\rho}}_m(t)$, where k_m is a constant defined as $k_m = \frac{(1-e_m)\rho_m}{\beta}$. Thus, the Lyapunov function is upper bounded by $V_s(t) \leq V_s(0) + k_m[\hat{\rho}_m(t) + \hat{\rho}_m(0)]$ on the basis of the comparison principle. Furthermore, on the condition that $\hat{\rho}_m(t) \leq \rho_m$, it is found that $V_s(t) \leq V_s(0) + 2k_m\rho_m$.

Based on the above analysis of these four cases, we prove that the adaptive gain $\hat{\rho}_m(t)$ is always upper bounded. Therefore, there always exists a positive scalar $\bar{\rho}_m$ such that $\hat{\rho}_m \leq \bar{\rho}_m$ and $\rho_m \leq \bar{\rho}_m$ for all $t > 0$. □

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