Brief paper

Saturated adaptive pose tracking control of spacecraft on SE(3) under attitude constraints and obstacle-avoidance constraints

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\begin{abstract}
In this paper, for a translation and rotation coupled spacecraft, a saturated adaptive pose controller is proposed to achieve the desired pose configuration tracking under the attitude constraints and obstacle-avoidance constraints, input saturation, and external disturbances. First, a set of relative kinematics and dynamics of a spacecraft on SE(3) under input saturation is established. Second, cancellable potential functions are designed for the attitude constraints and the obstacle-avoidance constraints by introducing the warning range. Then, based on the uniformly asymptotically stable theory of nonlinear vanishing perturbation systems, an integrated pose controller leveraging the cancellable potential functions is proposed for the spacecraft on SE(3) to achieve the desired pose configuration tracking under multiple constraints. Finally, the effectiveness of the proposed integrated pose controller is illustrated by a numerical simulation of a spacecraft on SE(3) tracking the desired pose configuration under multiple constraints.
\end{abstract}

\section{Introduction}

In recent years, space technology has developed rapidly with the increasing frequency of human space missions, such as on-orbit precision operation, rendezvous and docking (RVD), complex spacecraft formation flying, space target capture, deep space exploration and so on. There is a need for higher requirements on the reliability, accuracy and rapidity of the translation and rotation couple spacecraft control system to cope with the emergence of various complex their space missions.

The traditional methods of designing attitude controller and translational controller respectively are powerless in the face of high complexity and precision space missions, which affects the control effect. Therefore, it is of important research significance to devise the integrated modeling and control of spacecraft position and attitude. Previous studies have developed many modeling methods for rigid body pose (position and attitude) control, including the dual quaternions (Gui & Vukovich, 2016) and the Lie group SE(3) (Brás, Izadi, Silvestre, Sanyal, & Oliveira, 2016; Hamrah & Sanyal, 2022), as well as other rotation and position vector combination methods of Euler angles (Kristiansen, Nicklasson, & Gravdahl, 2008), quaternions (Sun & Huo, 2015) and modified Rodriguez parameters (Sun, Sun, & Jiang, 2022). The dual quaternions and the Lie group SE(3) are more compact than other methods. However, the fuzziness of the dual quaternions results in the same attitude being expressed by two different quaternions (Guo, Song, & Li, 2016). The pose model of spacecraft on SE(3) can avoid this problem, which has attracted extensive research recently (Dhullipalla, Hamrah, Warier, & Sanyal, 2019; Lee, Viswanathan, Holguin, Sanyal, & Butler, 2013; Zhang, Ye, Xiao, & Sun, 2022).

In addition, the spacecraft may be faced with the threat of collision with other spacecraft, targets and space obstacles (Hu, Dong, Zhang, & Ma, 2015). Meanwhile, the spaceborne equipment on the spacecraft needs to meet attitude constraints (Li, Wang, Zhang, & Duan, 2021). Recently, plenty of research efforts are devoted to the RVD of spacecraft, which considers both attitude constraints and obstacle-avoidance constraints. In Zappulla, Park, Virgili-Llop, and Romano (2019), considering the dynamic environment, the autonomous RVD of spacecraft was realized based on the artificial potential functions. In Henry, Zenteno-Torres, Cieslak, Ferreira De Loza, and Dávila (2021), a spacecraft modeled as a translation and rotation coupled model was established on dual quaternions, and a fault-tolerant controller was designed to realize the autonomous RVD. However, the above research only focuses on the special scene of spacecraft RVD, and cannot be...
applied to spacecraft pose control under attitude constraints and obstacle-avoidance constraints.

To the best of our knowledge, for the spacecraft with pose constraints, nonlinear input saturation, and external disturbances, designing an integrated pose tracking controller on SE(3) is still an open problem. We address these challenging issues by establishing relative kinematics and dynamics of the spacecraft on SE(3) firstly. To handle the actuators saturation, an input saturation model is constructed with a dead-zone based operation. Then, cancellable potential functions (the potential functions are cancelled or does not work when the spacecraft reaches the desired pose configuration) for the attitude constraints and the obstacle-avoidance constraints are designed by introducing the warning range. Finally, based on the uniformly asymptotically stable theory of nonlinear vanishing perturbations systems (Khalil, 2002, Section 9.1), an saturated adaptive pose (SAP) controller using the cancellable potential functions is proposed for the spacecraft to achieve the desired pose configuration tracking under multiple constraints.

The main contributions of this work are summarized as:
1. In contrast to the existing attitude potential functions in Chen and Shan (2021), Kulamani and Lee (2017) and Shen, Yue, Goh, Wu, and Wang (2018), the proposed attitude potential function, considers not only the static attitude-forbidden zones, but also the dynamic attitude-forbidden zones. In addition, we also proposed a method for calculating the warning angle and warning distance and applied these to the design of potential functions.
2. Compared with Huang, Yan, and Zhou (2017), Huang, Yan, Zhou, and Yang (2017) and Zhang, Ye, Biggs, and Sun (2019), the uniformly asymptotically stable theory of nonlinear vanishing perturbation systems is rigorously applied for the closed-loop stability analysis, where we obtain a conservative condition for satisfying the key requirement in the theory by using saturation operation.

2. Preliminaries

2.1. Kinematics and dynamics of a spacecraft on SE(3)

In this paper, a translation and rotation coupled spacecraft is considered. Let \( T(x, y, z) \) denote the Earth centered inertial (ECI) frame and \( B(x, y, z) \) denote the body-fixed frame with the origin being located at the centroid. Let \( p \in \mathbb{R}^3 \) express the position vector of the spacecraft in the ECI frame \( T \) and \( R \in SO(3) \) express the spacecraft rotation matrix from body-fixed frame \( B \) to ECI frame \( T \). The rotation matrix \( R \in SO(3) \) is a special Euclidean group used to parameterize attitude and is defined as (Yue et al., 2022) \( SO(3) = \{ R \in \mathbb{R}^{3 \times 3} | R^T R = I_3, \det(R) = 1 \} \). Then, a Lie group \( SE(3) \) can be represented by the semidirect product \( SE(3) = \mathbb{R}^3 \ltimes SO(3) \), and an element \( g \in \mathbb{R}^{4 \times 4} \) of the Lie group \( SE(3) \) and the corresponding inverse matrix \( g^{-1} \in \mathbb{R}^{4 \times 4} \) with the form

\[
g = \begin{bmatrix} R & p \\ 0_{1 \times 3} & 1 \end{bmatrix}, \quad g^{-1} = \begin{bmatrix} R^T & -R^T p \\ 0_{1 \times 3} & 1 \end{bmatrix},
\]

(1)

can be used to compactly represent the pose configuration of a spacecraft (Lee & Vukovich, 2016). The kinematics of a spacecraft can be given as (Brás et al., 2016)

\[
\dot{g} = g \dot{\xi}, \text{ with } \dot{\xi} = \begin{bmatrix} \omega^x & v^x \\ 0_{1 \times 3} & 0 \end{bmatrix} \in \mathbb{R}^{4 \times 4}.
\]

(2)

where \( \dot{\xi} \in \mathfrak{se}(3) \) is the isomorphism from vector space to Lie algebra associated with \( SE(3) \), \( \xi = [\omega^x, v^x]^T \in \mathbb{R}^6 \). \( \omega \in \mathbb{R}^3 \) and \( v \in \mathbb{R}^3 \) are the angular and the translational velocity vector of the spacecraft with respect to the ECI frame \( T \) and expressed in the body-fixed frame \( B \), respectively. \( \dot{\xi}^T \) is used to convert a vector in \( \mathbb{R}^3 \) to a \( 3 \times 3 \) skew-symmetric matrix (cf. Kang, Shen, Wu, & Damaren, 2023, Eq. (3)).

Then, the translational and rotational equations of a spacecraft are given by (Mei, Liao, Gong, & Luo, 2022)

\[
\dot{\xi} = \begin{bmatrix} \omega^x \\ v^x \end{bmatrix}, \quad \dot{\xi} = \begin{bmatrix} \dot{\theta}, \dot{\phi}, \dot{\psi} \end{bmatrix} = \begin{bmatrix} \omega^x, v^x \end{bmatrix}^T = \begin{bmatrix} \omega^x, v^x \end{bmatrix}^T \in \mathbb{R}^6,
\]

(3)

with

\[
\Omega = \text{diag}(J, m I) \in \mathbb{R}^{6 \times 6}, \quad \Gamma_B(\Omega) = [M_B^T, \mathbf{F}_B] \in \mathbb{R}^6,
\]

(4)

where \( m \in \mathbb{R}^{3 \times 3} \) denote the mass and the symmetric positive-definite inertia matrix. \( \mathbf{M}_B \in \mathbb{R}^3 \) and \( \mathbf{F}_B \in \mathbb{R}^3 \) are the gravity-gradient moment and the gravity force on the spacecraft. \( \mathbf{d}_R \in \mathbb{R}^3 \) and \( \mathbf{d}_t \in \mathbb{R}^3 \) are the external disturbance torque and force. \( \mathbf{u} \in \mathbb{R}^1 \) and \( \mathbf{f} \in \mathbb{R}^1 \) denote the control torque and force. The adjoint operator \( \text{ad}_d \) and the co-adjoint operator \( \text{ad}^*_d \) of \( \xi \) are expressed as

\[
\text{ad}_d = \begin{bmatrix} \omega^x & 0_{1 \times 3} & -v^x \\ v^x & 0_{1 \times 3} & \omega^x \end{bmatrix}, \quad \text{ad}^*_d = \begin{bmatrix} -\omega^x & -v^x \\ \omega^x & v^x \end{bmatrix} \in \mathbb{R}^{3 \times 3}.
\]

(5)

Considering the \( J_2 \) perturbation of the Earth’s oblateness and the coupling between translational and rotational motion, \( \mathbf{M}_B \) and \( \mathbf{F}_B \) in the body-fixed frame \( B \) are expressed as (Lee & Vukovich, 2016, Eqs. (5)–(7)). We assume that a virtual leader spacecraft flies in the absence of external disturbances and control. The subscript \( d \) indicates that the parameter is related to the virtual leader spacecraft. Thus, the virtual leader spacecraft is expressed as

\[
\begin{cases} \dot{\mathbf{g}}_d = \mathbf{g}_d \dot{\mathbf{g}}_d, \\
\Xi_d = \text{ad}^*_d \Xi_d \mathbf{g}_d + \Gamma_B(\Xi_d),
\end{cases}
\]

(6)

where the definitions of parameters are similar to (2) and (3). When the initial orbit and attitude parameters of the virtual leader spacecraft are known, (6) produces a desired tracking pose for the controllable spacecraft.

2.2. Relative kinematics and dynamics of a spacecraft on SE(3) under input saturation

Let \( g_\tau \in \mathbb{R}^{4 \times 4} \) denotes the relative error between the pose of the virtual leader spacecraft and the pose of the controllable spacecraft, can be expressed as (Mei et al., 2022)

\[
g_\tau = g_d^{-1}g = \begin{bmatrix} R_\tau^R & R_\tau^p(p - p_d) \\ 0_{1 \times 3} & 1 \end{bmatrix} = \begin{bmatrix} R_\tau & p_d \\ 0_{1 \times 3} & 1 \end{bmatrix}.
\]

(7)

Then, by the exponential coordinates, the pose configuration error \( g_\tau \) can be uniformly expressed as (Zhang, Biggs, Ye, & Sun, 2019)

\[
\eta = \begin{bmatrix} \Phi & \Psi \end{bmatrix} = \begin{bmatrix} \text{log}_{SE(3)}(g_\tau) \end{bmatrix}^T = \begin{bmatrix} \Phi^T & \Psi^T \end{bmatrix} = \begin{bmatrix} \Phi^T \end{bmatrix} \in \mathbb{R}^6,
\]

(8)

where \( \cdot^T \) is the operation of mapping the Lie algebra to the corresponding vector. \( \Phi \in \mathbb{R}^3 \) and \( \Psi \in \mathbb{R}^3 \) are the attitude and the position tracking error, respectively, which are expanded as (Zhang, Biggs, et al. (2019, Eq. (15) and Eq. (16)));

The relative velocity error \( \xi_v = [\omega_v, v_v]^T \in \mathbb{R}^6 \) in the body-fixed frame \( B \) of the controllable spacecraft is (Lee, Sanyal, & Butter, 2015)

\[
\xi_v = \xi - \text{Ad}_{\xi}\xi_d.
\]

(9)

In addition, the adjoint matrix mapping operation of the Lie group \( SE(3) \) is introduced as Zhang, Biggs, et al. (2019, Eq. (18)).
Then, the relative kinematics of a spacecraft using exponential coordinates can be represented as
\[
\dot{\xi} = G(q)\xi_v,
\]
where the definitions of \(G(q)\) is expressed in Zhang, Biggs, et al. (2019, Eq. (20a)). Then, taking the dynamics (3) into the time derivative of (9) and using the fact in Lee and Vukovich (2016, Appendix), the relative dynamics of the spacecraft are expressed as
\[
\dot{\xi}_v = H + \Xi^{-1}\Gamma_c + \Xi^{-1}R \xi + \Gamma_D,
\]
where \(H = \Xi^{-1}a_\xi^T \Xi \xi + a_\xi U \xi - A_{\xi\xi} \xi - A_{\xi\theta} \theta \). 

**Assumption 1.** The unknown disturbance \(\Gamma_D\) of the spacecraft is bounded by an unknown positive constant \(D_{\text{max}}\), i.e., \(\|\Gamma_D\| \leq D_{\text{max}}\).

In addition, the actuator saturation is also considered. The saturated control input \(\Gamma_c = \{\Gamma_{c,1}, \ldots, \Gamma_{c,6}\}\) in (11) is defined as \(\Gamma_{c,i} = \text{sign}(U_{c,i})\min(U_{\text{sat},i}, [U_{c,i}])\) (Kang et al., 2023), where \(U_{c,i}\) and \(U_{\text{sat},i}\) are the nominal input and saturation limit of the ith actuator of the spacecraft with \(i = 1, \ldots, 6\). The nonlinear saturation \(\Gamma_c\) in this work is approximately modeled as \(\tilde{\Gamma}_c = [\tilde{\Gamma}_{c,1}, \ldots, \tilde{\Gamma}_{c,6}]^T \in \mathbb{R}^6\) by using a dead-zone based model (Mousavi & Khayatian, 2011; Yue et al., 2023) with the relation
\[
\tilde{\Gamma}_{c,i} = \rho_{0,c} U_{c,i} - \int_{0}^{K_{\rho,i}} \rho_{c,i}(k) z(k, U_{c,i}) dk,
\]
where \(\rho_{c,i}(k)\) is a known density function and is given as \(\rho_{c,i}(k) = \begin{cases} \frac{1}{K_{\rho,i}} & r \leq K_{\rho,i} \\ 1 & r > K_{\rho,i} \end{cases}\) the dead-zone operator \(z(k, U_{c,i})\) is defined as \(z(k, U_{c,i}) = \max(U_{c,i} - k, \min(0, U_{c,i} + k))\). Meanwhile, \(\rho_{c,i} = \int_{0}^{K_{\rho,i}} \rho_{c,i}(k) dk\) is a positive constant parameter. According to Kang et al. (2023, Eq. (9)), we further have \(U_{\text{sat},i} = K_{\rho,i}\) from \(\rho_{c,i}\).

Then, the relative dynamics (11) can be rewritten as
\[
\dot{\xi}_v = \rho_{0,c} U_c - L_c,
\]
with \(\dot{\Gamma}_c = \rho_{0,c} U_c - L_c\),

where \(\rho_{0,c} = [\rho_{0,c,1}, \ldots, \rho_{0,c,6}]^T \in \mathbb{R}^6\), \(L_c = [l_{c,1}, \ldots, l_{c,6}]^T \in \mathbb{R}^6\) with \(l_{c,i} = \int_{0}^{K_{\rho,c,i}} \rho_{c,i}(k) z(k, U_{c,i}) dk, i = 1, \ldots, 6\), \(U_c = [U_{c,1}, \ldots, U_{c,6}]^T \in \mathbb{R}^{6}\) represents the controller output to be designed and the symbol \(\circ\) denotes Hadamard product (Horn, 1990).

### 2.3. Attitude and obstacle-avoidance constraints

In this subsection, the models of attitude constraints and obstacle-avoidance constraints are given.

#### 2.3.1. Attitude constraint

As shown in Fig. 2(a), for the spacecraft, \(a_m\) is the centerpointing unit vector in the body-fixed frame \(B\) of the \(m\)th \((m = 1, 2, \ldots, M)\) sensitive spaceborne equipment. There are \(N\) bright objects corresponding to the \(m\)th sensitive spaceborne equipment, which is located at position \(p_m \in \mathbb{R}^3\) in the ECI frame \(I\). The half cone angle of the field of view of the \(m\)th sensitive spaceborne equipment to the \(n\)th bright object is represented by \(\theta_m^n = \frac{\pi}{2} - \frac{\pi}{4} < \pi\).

In this work, when \(\|p_m\| > 100\|p\|\) is satisfied, the relative motion between the bright object and the spacecraft can be ignored. The corresponding attitude-forbidden zone is defined as the static attitude-forbidden zone (SAFZ). When another kind of bright object is close to the spacecraft, the relative motion between them cannot be ignored. The corresponding attitude-forbidden zone is defined as the dynamic attitude-forbidden zone (DAFZ). Then, the static/dynamic attitude-forbidden zone of the \(m\)th sensitive spaceborne equipment and the \(n\)th bright object can be uniformly expressed as
\[
a_m^T R_l^T \frac{p_m}{\|p_m\|} < \cos(\theta_m^n),
\]
with \(l_n = \begin{cases} \frac{p_m}{\|p_m\|} & \text{if } \|p_m\| \geq 100\|p\|, \\ \frac{p_m}{\|p_m\|} & \text{otherwise.} \end{cases}\)

In this work, the SAFZ is for distant bright objects (e.g., the Sun). The DAFZ is for neighboring bright objects (e.g., flares generated by multiple engines of adjacent non-cooperative spacecraft). We also assume that all luminous objects are considered to be bright spheres.

#### 2.3.2. Obstacle-avoidance constraint

In this work, obstacle-avoidance between the spacecraft and \(Q\) space obstacles (space debris or non-cooperative neighbor spacecraft) is considered. Assuming that the positions of all space obstacles are known, the \(q\)th space obstacle is located at position \(q = 1, 2, \ldots, Q\), with \(d_q^s = \|p - p_q\| > d_q, q = 1, 2, \ldots, Q\),

where \(d_q\) represents the minimum distance between the spacecraft and the \(q\)th obstacle. \(d_q^s\) is the relative distance between the spacecraft and the \(q\)th obstacle.

**Assumption 2.** The desired pose \(g_d\) from the virtual leader spacecraft and the initial pose \(g(t = 0)\) of the spacecraft meet the mixed attitude constraints (14) and obstacle-avoidance constraints (16).

### 3. Problem statement

The objective of this paper is to design a SAP controller for the spacecraft on \(SE(3)\) to achieve the desired pose configuration \(g_d\) tracking in the presence of attitude constraints and obstacle-avoidance constraints, input saturation, and disturbances, as shown in Fig. 1.

This work mainly solves the following two problems:

**Problem 1 (Cancellable Potential Function).** Considering the constraints composed of multiple static/dynamic attitude-forbidden zones and multiple space obstacles, propose cancellable potential functions that can disappear at the desired pose configuration \(g_d\).

**Problem 2 (Saturated Adaptive Pose Controller).** Based on the uniformly asymptotically stable theory of nonlinear vanishing
perturbation systems, design an SAP controller leveraging the cancellable potential function to achieve the desired pose configuration $g_d$ tracking under the multiple constraints.

4. Cancellable potential function

In this section, we solve Problem 1. Suppose the spacecraft is with $M$ sensitive spaceborne instruments, and there are $N_1$ bright objects (at long distance) corresponding to the $m$th ($m = 1, 2, \ldots, M$) equipment. Meanwhile, there are $Q$ space obstacles around the orbit of the virtual leader spacecraft, and each space obstacle is assumed to be a bright object at a close distance.

4.1. Attitude cancellable potential function

There are $M$ sensitive spaceborne instruments, which is required to avoid exploring directly to both $N_1$ bright objects at long distance and $Q$ space obstacles at close distance. Therefore, there are $N = N_1 + Q$ static/dynamic attitude-avoided forbidden zones for the $m$th ($m = 1, 2, \ldots, M$) equipment. To further protect the sensitive equipment, we define an warning angle $\theta_m^w < \sigma_m^w < \pi$ for the $m$th ($n = 1, 2, \ldots, N$) attitude-avoided forbidden zone.

**Assumption 3.** The desired pose configuration $g_d$ makes the center pointing vector of the $m$th sensitive spaceborne equipment outside the $n$th bright object attitude warning zone, i.e., $(a_m, R_I^n) > \sigma_m^n$, where $(a, b)$ is the angle of vectors $a$ and $b$.

Then, motivated by Kang, Shen, and Wu (2020, Definition 2) and based on attitude constraint model (14) and Assumption 3, the attitude cancellable potential function $U_m^n$ is constructed as

$$ U_m^n = \begin{cases} \left( \frac{(a_m^m R_I^n - \cos(\theta_m^n))^2}{(\cos(\theta_m^n) - a_m^m R_I^n)} \right)^2, & \theta_m^n < (a_m, R_I^n) \leq \sigma_m^n, \\ \sigma_m^n < (a_m, R_I^n), & 0, \end{cases} $$

where $m = 1, \ldots, M$, $n = 1, \ldots, N$. $\sigma_m^n$ is the $n$th warning angle of the $m$th sensitive equipment.

As observed in (17), the attitude cancellable potential function $U_m^n$ works only when the $m$th sensitive spaceborne equipment center pointing vector $a_m$ is located in the warning zone of the $n$th attitude-forbidden zone.

4.2. Obstacle-avoidance cancellable potential function

Suppose that the $q$th ($q = 1, 2, \ldots, Q$) space obstacle and the spacecraft are in the spherical envelope with radii $r_q$ and $r_s$, respectively, as shown in Fig. 2(b). Thus, the minimum distance $d_q = r_q + r_s$ in (16) between the spacecraft and the $q$th space obstacle. Then, we define the warning distance $d_q^w$ ($d_q^w > d_q$) for the spacecraft.

**Definition 1.** [Warning distance] As shown in Fig. 2(b), suppose that the spacecraft with mass $m$ is approaching the $q$th space obstacle with its maximum allowable relative speed $v_{\text{max}}$. The initial relative position of the $q$th space obstacle to the spacecraft is $d_q^0$. Suppose that the maximum force $F_{\text{max}}$ is applied to decelerate the spacecraft, the final relative speed of the spacecraft reduces to zero when the relative distance of the $q$th space obstacle and the spacecraft is $d_q^f = d_q$. The warning distance of the spacecraft and the $q$th space obstacle is $d_q^w = d_q^0 - d_q^f + d_q^w > d_q^0$. Then, the spherical zone with the warning distance $d_q^w$ as the radius is regarded as the collision warning zone between the spacecraft and the $q$th space obstacle.

According to Definition 1, we have $d_q = d_q^w + 1/2 \max \{ \mu^2 / v_{\text{max}}^2 + d_q^w \}$. Then, we can obtain a unique solution to $\mathbf{f}$ as well. Thus, the warning distance $d_q^w$ can be defined as $d_q^w = 1/2 \max \{ \mu^2 / v_{\text{max}}^2 + d_q^w \}$.

**Assumption 4.** The desired pose configuration $g_d$ makes the $q$th space obstacle outside the $q$th collision warning zone of the spacecraft, i.e., $\| p_q - p_q^* \| > d_q^w$.

Then, according to the obstacle-avoidance constraint model (16) and Assumption 4, the obstacle-avoidance cancellable potential function $F_q$ is given as

$$ F_q = \begin{cases} \left( \frac{(d_q^w - p_q - p_q^*)^2}{(p_q - p_q^*)^2} \right)^2, & d_q < \| p - p_q^* \| \leq d_q^w, \\ 0, & d_q^w < \| p - p_q^* \|. \end{cases} $$

The obstacle-avoidance cancellable potential function $F_q$ is only effective when the $q$th space obstacle enters the $q$th collision warning zone of the spacecraft.

From (17) and (18), the desired pose configuration $g_d$ is the equilibrium point of the attitude cancellable potential function $U_m^n$ and the obstacle-avoidance cancellable potential function $F_q$. To this end, this kind of potential function is defined as a cancellable potential function.

5. Saturated adaptive pose controller

In this section, we solve Problem 2. In order to design the controller, a sliding mode vector $s \in \mathbb{R}^6$ is constructed as $s = \xi + \nu \eta$, where $\nu = \text{diag}(\nu_1, \nu_2, \ldots, \nu_{16}) \in \mathbb{R}^{16 \times 16}$ is positive-definite diagonal matrix with positive diagonal elements. According to (10) and (13) of the spacecraft, the time derivative of $s$ is

$$ \dot{s} = H + \Xi^{-1} F_c + \Xi^{-1} F_g(\xi) + F_D + \nu \eta G(q) \xi. $$

Noted that the upper bound $D_{\text{max}}$ of the disturbances $F_D$ is limited by a known constant, thus the estimation $\hat{D}_{\text{max}}$ of the parameter $D_{\text{max}}$ is also limited to a known bounded convex set. This can be realized by using a smooth projection algorithm (Thakur, Srikant, & Akella, 2015) to modify the parameter update law. Two convex sets are defined as

$$ \Omega_{D_{\text{max}}} = \{ D_{\text{max}} \in \mathbb{R}^+ | D_{\text{max}} < \epsilon \}, $$

$$ \Omega_{\hat{D}_{\max}} = \{ \hat{D}_{\text{max}} \in \mathbb{R}^+ | \hat{D}_{\text{max}} < \epsilon + \delta \}, $$

where $\epsilon > 0$ and $\delta > 0$ are known constants. The smooth projection-based update law for $\hat{D}_{\text{max}}$ is given by

$$ \hat{D}_{\text{max}} = \text{Proj}(\hat{D}_{\text{max}}, \gamma), \quad \gamma = \lambda_{\text{max}}(\Xi) \| s \|, $$

where $\text{Proj}(\hat{D}_{\text{max}}, \gamma) \triangleq \begin{cases} \beta \gamma, & \bar{D}_{\text{max}}^2 < \epsilon, \\ \beta \left( \gamma - \frac{(\bar{D}_{\text{max}} - \gamma)^2}{2 \bar{D}_{\text{max}}^2} \right) \delta \bar{D}_{\text{max}} D_{\text{max}} \left( \hat{D}_{\text{max}} \right) \end{cases}$ if $\bar{D}_{\text{max}}^2 \geq \epsilon$.
Then, according to (19) and the projection-based update law for $D_{\max}$ in (21), a basic controller is designed as

$$U_c = \chi \left( - \Xi(\mathbf{H} + \mathbf{V}_1 \mathbf{G}(\mathbf{\xi}^*) - \Gamma_{\gamma}(\Xi) \right) - k_1 \mathbf{s} - \frac{\lambda_{\max}(\Xi) \tilde{D}_{\max} \mathbf{s}}{\|\mathbf{s}\|} + L_c, \tag{22}$$

where $k_1 = \text{diag}(k_{11}, \ldots, k_{16}) \in \mathbb{R}^{6 \times 6}$ is a positive-definite diagonal matrix, \( \chi = \frac{1}{\rho_{0,c,1}} \ldots \frac{1}{\rho_{0,c,6}} \in \mathbb{R}^{6} \), and the updating laws for the adaptive parameter $\kappa$ is proposed as $\kappa = \frac{\mathbf{k}^T}{\mathbf{y}^T}$. Using the proposed basic controller $U_c$ in (22), we have the following theorem.

**Theorem 1.** For the spacecraft expressed by (10) and (13), the proposed basic controller (22) ensures that $\lim_{t \to \infty} \mathbf{\xi}(t) = 0$ and $\lim_{t \to \infty} \dot{\mathbf{\xi}}(t) = 0$.

**Proof.** Consider the Lyapunov function $V_1 = \frac{1}{2} \mathbf{s}^T \mathbf{S} \mathbf{s} + \mathbf{V}_\text{max} + \mathbf{V}_s$, where $\mathbf{V}_\text{max} = \frac{1}{2} \mathbf{D}_{\max}^2$ and $\mathbf{V}_s = \frac{1}{2} \mathbf{s}^T \mathbf{D}_{\max} \mathbf{s}$. Substituting (19) and the proposed basic controller (22) into the time derivative of $V_1$ yields

$$\dot{V}_1 \leq -\min(k_1) \mathbf{s}^T \mathbf{S} \mathbf{s} - \lambda_{\max}(\Xi) \|\mathbf{s}\| \tilde{D}_{\max} \mathbf{s} + \lambda_{\max}(\Xi) \|\mathbf{s}\| \tilde{D}_{\max} - \frac{1}{\beta} \tilde{D}_{\max} \mathbf{s} \leq -\min(k_1) \mathbf{s}^T \mathbf{S} \mathbf{s} - \lambda_{\max}(\Xi) \|\mathbf{s}\| \mathbf{c} \leq -\min(k_1) \mathbf{s}^T \mathbf{S} \mathbf{s} - \mathbf{c},$$

where $\mathbf{c} = -\frac{\mathbf{D}_{\max} \mathbf{s}}{\|\mathbf{s}\|} \beta \lambda_{\max}(\Xi) \|\mathbf{s}\|$ and $\min(\cdot)$ is the minimum element.

According to (21), $c = 0$ if $\mathbf{D}_{\max}^2 < \epsilon$. In addition, when $\mathbf{D}_{\max}^2 \geq \epsilon$, $c = \frac{(\mathbf{D}_{\max} \mathbf{s})^2}{\epsilon} \frac{\mathbf{D}_{\max} \mathbf{s}}{\|\mathbf{s}\|} \mathbf{D}_{\max} \mathbf{s} \leq 0$ due to $\mathbf{D}_{\max} \mathbf{s} = \mathbf{D}_{\max} \mathbf{s}$. Therefore, $\dot{V}_1 \leq -\min(k_1) \mathbf{s}^T \mathbf{S} \mathbf{s}$ since $\min(k_1) > 0$. By invoking Barbalat’s Lemma (Khalil, 2002, Section 8.3), it yields that $\lim_{t \to \infty} \mathbf{s}(t) = 0$. Then, consider the Lyapunov function $V_2 = \frac{1}{2} \mathbf{q}^T \mathbf{q}$, substituting (10) and $\mathbf{\xi}_s = -\mathbf{v}_1 \mathbf{q}$ into the time derivative of $V_2$ further yields $\dot{V}_2 = -\mathbf{v}_1 \mathbf{q}^T \mathbf{G}(\mathbf{q}) \mathbf{q}$ where the fact $\mathbf{G}(\mathbf{q}) = \mathbf{q}$ (Lee et al., 2015) is used. Then, based on Barbalat’s Lemma, the pose error $\mathbf{q} = 0$ as $t \to \infty$. When $\mathbf{q} = 0$, we can get $\dot{\mathbf{q}} = 0$, and then substitute it into (10) to further obtain $\dot{\mathbf{q}} = 0$ due to $\mathbf{G}(\mathbf{q}) \neq 0$. Thus, it is clear that $\lim_{t \to \infty} \mathbf{q}(t) = 0$ and $\lim_{t \to \infty} \dot{\mathbf{q}}(t) = 0$.

This completes the proof.

Then, the force and torque produced by the cancellable potential functions (17) and (18) can be expressed as

$$U_{PF} = \left[ -k_3 \sum_{m=1}^{N} \sum_{n=1}^{\mathbf{m}} \frac{\partial U_m}{\partial (\mathbf{R}_{mn})} - k_4 \mathbf{R}^T \sum_{q=1}^{\mathbf{r}} \frac{\partial F_q}{\partial p} \right]^T. \tag{24}$$

Then, the SAP controller is designed as

$$U^*_c = \mathbf{U}_c + \chi \circ \mathbf{U}_{PF}, \tag{25}$$

where $\mathbf{U}_c = \rho_{0,c} \circ \mathbf{U}_c - L_c \in \mathbb{R}^6$ is the ith actuator saturation value $K_{\text{sat},i} = K_{i,c}$, and $\mathbf{U}_{PF} = \rho_{0,PF} \circ \mathbf{U}_{PF} - L_{PF} \in \mathbb{R}^6$ is the dead-zone based saturation operation for the control force/torque (24).

In this work, we assume that the saturated value $K_{\text{sat},i} = 4K_{i,c}$ is given. This setting is for $\|\mathbf{U}_c\| < \|\chi \circ \mathbf{U}_{PF}\|$ to achieve that the potential function plays a leading role when it is necessary to ensure the safety of the spacecraft. The selection $K_{\text{sat},i} = 4K_{i,c}$ is not unique, and suitable coefficients can be determined through multiple simulations.

Then, the SAP controller (25) is substituted into the time derivative of (19) to further obtain

$$\dot{s} = f(\mathbf{t}, s) + h(\mathbf{t}, s), \tag{26}$$

where $f(\mathbf{t}, \mathbf{s})$ and $h(\mathbf{t}, \mathbf{s}) = \Xi^{-1} \mathbf{U}_{PF}$ represent the nominal system and the perturbation term, respectively, and $f(\mathbf{t}, \mathbf{s}) = \mathbf{H} + \Xi^{-1}(\rho_{0,c} \circ \mathbf{U}_c - L_c) + \Xi^{-1} \Gamma_{\gamma}(\Xi)$ where $\rho_{0,c} \circ \mathbf{U}_c - L_c = \rho_{0,c} \circ \mathbf{U}_c - L_c$ is applied.

According to Theorem 1, when $\lim_{t \to \infty} \mathbf{s}(t) = 0$, the spacecraft reaches the equilibrium point. At this time, according to (17) and (18), the perturbation term $h(\mathbf{t}, \mathbf{s}) = 0$. Therefore, the system (26) is a vanishing perturbation system. Thus, the stability of the vanishing perturbation system (26) is summarized as

**Theorem 2.** When the positive vector $k_1$ is selected to be sufficiently large, the origin $\mathbf{s} = 0$ is uniformly asymptotically equilibrium point of the perturbed system (26).

**Proof.** By choosing the same Lyapunov function $V_1$ as in Theorem 1, we obtained $\frac{\dot{V}_1}{\mathbf{D}_{\max} \mathbf{s}} \leq -\min(k_1) \|\mathbf{s}\|^2$, and $\|\mathbf{D}_{\max} \mathbf{s}\| \leq \lambda_{\max}(\Xi) \|\mathbf{s}\|$ by following the same procedures as in Theorem 1.

Then, the derivative of $V_1$ along the trajectories of (26) is given by $\dot{V}_1 \leq -\min(k_1) \|\mathbf{s}\|^2 + \frac{\lambda_{\max}(\Xi)}{\|\mathbf{s}\|} \|\mathbf{t}_c \| \|\mathbf{t}_s\|$. Therefore, when the inertial parameters $\Xi$ of the spacecraft are determined, the larger the minimum element of the vector $k_1$, the more conducive to the establishment of (27).

In this work, after introducing the saturation operation, the term $\|h(\mathbf{t}, \mathbf{s})\|$ is bounded rather than tending to infinity in the aforementioned works. Based on this, we obtain conservative conditions for the establishment of inequality (27), which is a major contribution of this work.

**Remark 1.** Recalling the following previous works:

1. In Huang, Yang, and Zhou (2017, Theorem 2, Eq. 59), it is not strict to directly assume that inequality (27) is satisfied.
2. In Huang, Yang, Zhou, and Yang (2017, Theorem 1, Eq. (69)) and Zhang, Ye, et al. (2019, Theorem 2, Eq. (39)), the potential function in the inequality of the perturbation term and the equilibrium point tends to infinity, thus it is difficult to find the proper parameters to satisfy the inequality relation.

In this work, after introducing the saturation operation, the term $\|h(\mathbf{t}, \mathbf{s})\|$ is bounded rather than tending to infinity in the aforementioned works. Based on this, we obtain conservative conditions for the establishment of inequality (27), which is a major contribution of this work.
Remark 2. According to the adaptive law (21), a larger $\beta$ may cause oscillations, while a smaller $\beta$ may slow down the stabilization speed of the adaptive parameter $\text{Proj}(D_{\text{max}}, \gamma)$. Moreover, for parameters in basic controller (22), a larger $k_1$ may lead to excessive system damping and slower convergence speed, whereas a smaller $k_1$ may cause unnecessary oscillations. Meanwhile, a larger $k_1, V_1$ may accelerate the convergence speed of the system but cause unnecessary oscillations. Finally, larger $k_5$ and $k_6$ in (24) may lead to severe oscillations when the spacecraft responds to pose constraints, while smaller $k_5$ and $k_6$ may cause violation of the pose constraints.

6. Simulation results

In this section, the effectiveness of the proposed SAP controller (25) is verified by a numerical simulation.

The Orbital Elements of the virtual leader spacecraft are provided to the same as Kang et al. (2022). The inertia parameters of the two spacecraft and the input limits of the attitude actuators and the translation actuators are set to the same as Mei et al. (2022). The initial pose configuration $g_d(0)$ and velocity $\xi_d(0)$ of the virtual leader spacecraft are

$$g_d(0) = \begin{bmatrix} 0.3526 & -0.64 & 0.6827 & -1.1991 \times 10^6 \\ -0.64 & 0.3674 & 0.6749 & 6.1442 \times 10^6 \\ -0.6827 & -0.6749 & -0.28 & 2.8105 \times 10^6 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\xi_d(0) = 10^3 \times [0, 0, 0, -2.4746, 2.1108, -6.9752]^T,$$

where the units of position vector, translation velocity vector and angular velocity vector are m, m/s and rad/s, respectively. Then, the initial attitude $R(0)$ and the initial angular velocity $\omega(0)$ of the spacecraft are

$$R(0) = \begin{bmatrix} 0.5723 & -0.8138 & -0.1010 \\ 0.8138 & 0.5484 & 0.1922 \\ -0.1010 & -0.1922 & 0.9761 \end{bmatrix}, \omega(0) = [0, 0, 0]^T.$$

In the body-fixed frame $\mathcal{B}_S$ of the virtual leader spacecraft, the relative position and relative velocity of the spacecraft and the virtual leader spacecraft are given as $[90, -2000, 80]^T$ m and $[1, -0.5, 1]^T$ m/s.

The unit vector of a sensitive spaceborne equipment on the body-fixed frame $\mathcal{B}$ is $\mathbf{a} = [0, 0, 1]^T$. In addition, three SAFZ in the ECI frame $\mathcal{I}$ are considered, which are the same as Kang et al. (2023, Table I). All three static forbidden angles are set to 18 deg. Three space obstacles near the orbit of the virtual leader spacecraft are considered, with relative positions $[60, -1800, 30]^T$ m, $[25 + 30 \sin(0.1t), -800 - 30 \cos(0.1t), 20 + 30 \sin(0.1t)]^T$ m, and $[-20, -550, 600]^T$ m. The spacecraft and three space obstacles are assumed to be within a spherical envelope with a radius of 15 m, thus $d_q = 30$ m. These three space obstacles are bright objects, forming three DAFFZ with 15 deg forbidden angle. The maximum angular translation velocity of the spacecraft are $\|\omega_v\|_{\text{max}} = 10$ deg/s and $\|\psi_v\|_{\text{max}} = 13$ m/s. Then, the warning angle and warning distance are $\gamma_1^0 = \frac{1}{10} \times \frac{25.5}{180} \times (\frac{10}{180})^2 + \theta_1^2 \approx \frac{15\pi}{180} + \theta_1^2$ rad and $d_w^n = \frac{1}{2} [10.10] \times 10^3 + d_q \approx 530 + d_q$ m for $n = 1, 2, \ldots, 6$. $\theta_1^2$ represents the forbidden angle of each attitude-forbidden zone. For SAP controller (25), $V_1 = \text{diag}(0.1, 0.1, 0.1, 0.08, 0.012, 0.08), \epsilon = 10^{-4}, \delta = 10^{-6}, \beta = 10^{-4}, D_{\text{max}}(0) = 0, k_3 = 10^{-3}, k(0) = 1, k_5 = 5, k_4 = 10^{-3}, k_1 = 100 \times \text{diag}(1, 1, 1, 1, 1, 1)$.

As shown in Fig. 3(a), the proposed SAP controller (25) achieves attitude tracking in 350 s with a steady-state error $\|\Psi\| \leq 5 \times 10^{-3}$ in 950 s. The maximum angular velocity tracking error $\|\omega_v\|_{\text{max}} \leq 10$ deg/s, and the steady-state error $\|\omega_v\| \leq 3 \times 10^{-3}$ deg/s in 950 s, as shown in Fig. 3(b). From Fig. 3(c), the input saturation limitation is satisfied by using the dead-zone saturation operation. From Fig. 4(a), the proposed SAP controller (25) achieves position tracking under obstacle-avoidance constraints, where the position tracking is completed in 600 s with a steady-state error $\|\Psi\| \leq 0.06$ m in 950 s. From Fig. 4(b), the maximum translation velocity tracking error is $\|\psi_v\|_{\text{max}} \leq 13$ m/s, and the translation velocity tracking error converges in 950 s with a steady-state error $\|\psi_v\| \leq 8 \times 10^{-4}$ m/s. From Fig. 4(c), it is clear that the saturation limit of control force is satisfied.

The angles between the pointing direction of the sensitive equipment and the central pointing of the attitude-forbidden
zones are shown in Figs. 5(a) and 5(b). The proposed SAP controller (25) ensures the safety of the sensitive equipment. Meanwhile, according to Fig. 5(c), the collision threat between the spacecraft and the obstacles is avoided. According to Fig. 5, the initial pose configuration and desired pose configuration of the spacecraft satisfy the required Assumptions 2–4. As seen from Fig. 6, the key condition (27) for vanishing disturbances is satisfied, which is an improvement compared with Huang, Yan, and Zhou (2017, Theorem 2, Eq. (59)), Huang, Yan, Zhou, and Yang (2017, Theorem 1, Eq. (69)), and Zhang, Ye, et al. (2019, Theorem 2, Eq. (39)).

7. Conclusions

In this paper, based on the uniformly asymptotically stable theory of nonlinear vanishing perturbation systems, a saturated adaptive pose controller is proposed for the spacecraft on SE(3) to realize the desired pose configuration tracking with the attitude constraints and obstacle-avoidance constraints, input saturation, and disturbances. Based on the dead-zone saturation operation, the relative kinematics and dynamics on SE(3) are constructed. Cancellable potential functions are designed by introducing the warning range for the pose constraints. Then, the SAP pose controller leveraging the cancellable potential functions is proposed to achieve the desired pose configuration tracking under pose constraints. Finally, simulation results demonstrate the efficiency of the proposed integrated pose controller. In future works, the acquisition of attitude information of space obstacles, the pointing deviation of spaceborn equipment, and the underactuated coupling control near Small Bodies will be explored.

References


Sun, Liang, & Sun, Guang (2022). Disturbance observer-based saturated fixed-time pose tracking for feature points of two rigid bodies. Automatica, 144, Article 110475.


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