Corrections to "Rigid-body attitude stabilization with attitude and angular rate constraints"

Some errors in [1] are corrected as follows.

In the proof of Theorem 1, we missed a term to derive Equation (22) from Equation (21). Therefore, we have to make some minor changes to the attitude controller in Equation (17) to ensure the stability of the overall attitude control system. In the following, we give these corrections in detail.

In the original paper, as stated in Equation (5), a sliding vector $\boldsymbol{s} = [s_1, s_2, s_3]^T \in \boldsymbol{\mathcal{R}}^3$ is designed as

$$\boldsymbol{s} = \boldsymbol{\omega} + k\boldsymbol{q}_e, \tag{1}$$

where k is a positive constant. To compensate the effects of the missing term in the original stability proof, we rewrite the attitude dynamics in terms of the sliding vector as

$$\boldsymbol{J}\dot{\boldsymbol{s}} = \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e, \boldsymbol{Q}, \boldsymbol{s}) - kk_2\boldsymbol{q}_e^T\text{Vec}[(\nabla V_a^* \otimes \boldsymbol{Q})]\frac{\boldsymbol{\Upsilon s}}{\|\boldsymbol{s}\|^2} + \boldsymbol{\tau} + \boldsymbol{d} \tag{2}$$

with

$$\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e, \boldsymbol{Q}, \boldsymbol{s}) = -\boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{J}\boldsymbol{\omega} + \frac{k}{2}\left(\boldsymbol{S}(\boldsymbol{q}_e) + q_{e0}\boldsymbol{I}_3\right)\boldsymbol{\omega} + kk_2\boldsymbol{q}_e^T\text{Vec}[(\nabla V_a^* \otimes \boldsymbol{Q})]\frac{\boldsymbol{\Upsilon s}}{\|\boldsymbol{s}\|^2}, \tag{3}$$

where $\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e, \boldsymbol{Q}, \boldsymbol{s})$ is a nonlinear term in Equation (2) of this document (the Eqaution (5) of the original paper), $\boldsymbol{\Upsilon} = \boldsymbol{J}\boldsymbol{\Psi}$[1], $\boldsymbol{\Psi} = \text{diag}\{(s_{1,\max}^2 - s_1^2), (s_{2,\max}^2 - s_2^2)\ , (s_{3,\max}^2 - s_3^2)\}$, k_2 is a positive constant. Then, the same adaptive attitude controller as presented in Equations (17)-(19) can be used to achieve the result in Theorem 1, except that the nonlinear feedback term $\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e)$ in Equation (17) is replaced by the foregoing new $\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e, \boldsymbol{Q}, \boldsymbol{s})$.

Next, in the stability analysis, the same Lyapunov function candidate V_ℓ in Equation (20) is selected. The time derivative of V_ℓ is given by

$$\begin{aligned}
\dot{V}_\ell =& 2kk_1\boldsymbol{q}_e^T\boldsymbol{\omega} + 2k_2\nabla V_a^T(\frac{1}{2}\boldsymbol{Q}\otimes\boldsymbol{\nu}(\boldsymbol{\omega})) + \boldsymbol{s}^T\boldsymbol{\Upsilon}^{-1}\boldsymbol{J}\dot{s} \\
&+ \frac{1}{\rho}(\hat{\bar{d}} - d_{\max})\dot{\hat{\bar{d}}} + \frac{\mu}{\delta}(\hat{d}_{\max} - d_{\max})\dot{\hat{d}}_{\max} \\
=& 2kk_1\boldsymbol{\omega}^T\boldsymbol{q}_e - k_2\boldsymbol{\omega}^T\text{Vec}[(\nabla V_a^* \otimes \boldsymbol{Q})] \\
&+ \boldsymbol{s}^T\boldsymbol{\Upsilon}^{-1}\left[\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e, \boldsymbol{Q}, \boldsymbol{s}) - kk_2\boldsymbol{q}_e^T\text{Vec}[(\nabla V_a^* \otimes \boldsymbol{Q})]\frac{\boldsymbol{\Upsilon s}}{\|\boldsymbol{s}\|^2} + \boldsymbol{\tau} + \boldsymbol{d}\right] \\
&+ \frac{1}{\rho}(\hat{\bar{d}} - d_{\max})\dot{\hat{\bar{d}}} + \frac{\mu}{\delta}(\hat{d}_{\max} - d_{\max})\dot{\hat{d}}_{\max}
\end{aligned} \tag{4}$$

where $\nabla V_a^T(\boldsymbol{Q}\otimes\boldsymbol{\nu}(\boldsymbol{\omega})) = -\boldsymbol{\omega}^T\text{Vec}[(\nabla V_a^* \otimes \boldsymbol{Q})]$ is used. Then, following the same lines as shown in the original paper, we can complete the proof.

References

[1] Qiang Shen, Chengfei Yue, Cher Hiang Goh, Baolin Wu, and Danwei Wang. Rigid-body attitude stabilization with attitude and angular rate constraints. *Automatica*, 90:157–163, 2018.

[1] In the original paper $\boldsymbol{\Upsilon}$ is defined as $\boldsymbol{\Upsilon} = \boldsymbol{\Psi}\boldsymbol{J}^{-1}$, which is not correct. The correct one is $\boldsymbol{\Upsilon} = \boldsymbol{J}\boldsymbol{\Psi}$.