Research of Aero-Engine Robust Fault-Tolerant Control Based on Linear Matrix Inequality Approach

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Abstract. Associated with the problem of fault tolerant control against actuator fault happened to the aero-engine, a D-stabilized robust tolerant control approach was proposed based on linear matrix inequality (LMI). To satisfy the regional poles constraints and performance in actuator fault condition and structured perturbations, a state feedback fault tolerant controller was designed in terms of feasible solutions to the LMIs. Simulation results illustrate that, the proposed approach can maintain the performance of robust stability in specified pole region as well as integrity of the control system in actuator fault condition, with allowable perturbations and fault bounds.

Keywords: aero-engine; actuator fault; linear matrix inequality; robust fault tolerant control.

1 Introduction

Stimulated by the growing demand for high reliability and high security of aero-engine control systems, robust fault-tolerant control has become a critical issue since its goal is to make the system stable and retain acceptable performance under the system faults. In comparison with the costly implementation of hardware redundancy, or on-line component cut which may lead to serious oscillation, the application of robust fault-tolerant control is a good deal by modifying the commands to the actuators and reconfiguring the control law. Nevertheless, the uncertainties between routine slight drift linearization model and turbine engine dynamics is also considered through the whole process of controller design, which highlights practical significance to engineering.

A broad class of sensor and actuator faults tolerant methods makes explicit use of a mathematical model of the plant. Due to analytical redundancy, which makes the reconfigured control law possible, the fault component can be compensated by the rest. Survey papers by [1] [2] present excellent overviews of advances in the research of fault-tolerant control. In [3]-[5] the authors studied sensor/actuator failure and derived tolerant methods via a iterative solution of Riccati matrix equations. However, some dynamic performance must be sacrificed to guarantee stability due to the short of optimizing undetermined parameters. Here the linear matrix inequality (LMI) process based on interior point method helps establish conditions of solvability of the problem. Then a set of feasible solutions are obtained as well as in the condition of multi-objective control (e.g., see the discussion in [6]-[8]).

In this paper, Section 2 discusses general form of turbine engine linear dynamic models, and transforms required performance into the solvability of LMI. Section 3 elaborates on designing a state feedback robust D-stabilized fault-tolerant controller with closed loop pole constraint, based on Lyapunov stability theorem and LMI approach. On occurrence of actuator failures, all closed loop system pole still allocated to a specified region, with allowable perturbations and fault bounds. Simulation of turbine engine control system is conducted and robust stability as well as dynamic performance are all illustrated in Section 4. Some corresponding conclusions are given in Section 5.

2 Problem Statement

To develop a robust fault-tolerant engine controller an accurate representation of the engine dynamics is desired. Although turbine engine dynamics are inherently nonlinear, aerodynamic nonlinearities and inertial coupling effects generally are smooth enough in the operating regions so that linear design techniques are applicable [9].

In this section a linear normalized state-space model of a certain type of turbofan engine around an operating point can be described as

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \tag{1}$$

where $x(t)=[n_L, n_H]^T, u(t)=[m_f, A_8]^T, A, B \in R^{2\times 2}$, and n_L、n_H、 m_f、A_8 respectively represent the fan rotor speed, the core rotor speed, the lord combustor fuel flow, and the exhaust nozzle area. Perturbation matrixes ΔA、 ΔB reflect the mismatch between original model structure and the real one when some condition changes in the system. Given that norm bounded perturbations takes the form

$$\begin{bmatrix}\Delta A & \Delta B\end{bmatrix} = E\Sigma(t)\begin{bmatrix}F_1 & F_2\end{bmatrix} \tag{2}$$

where E、 F_1 and F_2 are matrixes of proper dimensions and usually assumed to be known as uncertain structural information. $\Sigma(t)$ is unknown but with content $\Sigma^T(t)\Sigma(t) \le I$.

Suppose (A, B) is stabilizable, ensuring system asymptotic stability. With the state feedback control law

$$u(t) = Kx(t) \tag{3}$$

the state space model becomes

$$\dot{x}(t) = A_c x(t) \tag{4}$$

where $A_c = A + BK + \Delta A + \Delta BK$.

The eigenvalue of A_c is defined as $\lambda = \zeta\omega_n + j\omega_d$, and allocated to a circular region $D(r,\ q)$ of which centre is $(-q,\ 0)$ and radius r in the phase plane. It is equivalent to ensure minimum disturbance attenuation α and damping ratio $\zeta = \cos\theta$, as well as maximum damped oscillation frequency $\omega_d = r'\sin\theta$ of the system. By this means we can satisfy the required dynamics of overshoot, rise time and accommodation time. Such the circular region $D(r,\ q)$ is shown as Fig.1.

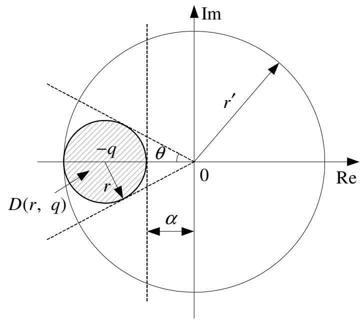

Fig. 1. Region $D(r, q)$

Lemma 1[6]. If all eigenvalues of matrix A lies in a circular region $D(r,\ q)$ of which centre is $(-q,\ 0)$ and radius r , $\sigma(A) \subset D$ in mathematical expression, matrix A is so-called D-stable, if and only if a symmetric positive definite matrix P exists, so that

$$\begin{bmatrix} -rP & qP + AP \\ qP + PA^T & -rP \end{bmatrix} < 0 \tag{5}$$

Suppose possible actuator fault model as

$$u_f(t) = Mu(t) \tag{6}$$

where the fault matrix is $M = diag\{m_1,\ m_2, \cdots, m_r\}$, $0 \le m_{il} \le m_i \le m_{iu} \le 1$ ($i = 1, 2, \ldots, r$) ,with each element corresponding to a specific fault. Here $m_i = 1$ indicates actuator i is in normal operation, $m_i = 0$ in total failure state, $0 < m_{il} \le m_i \le m_{iu} < 1$ partial deteriorated. Cite the following symbols:

$$M_0 = diag(m_{01},\ m_{02}, \cdots, m_{0r})$$
$$J = diag(j_1,\ j_2, \cdots, j_r)$$
$$L = diag(l_1,\ l_2, \cdots, l_r)$$

$m_{0i} = \dfrac{m_{il} + m_{iu}}{2}$, $j_i = \dfrac{m_{iu} - m_{il}}{m_{iu} + m_{il}}$, $l_i = \dfrac{m_i - m_{0i}}{m_{0i}}$. Easy to see that $M = M_0(I + L), |L| \le J \le I$.

Hence forms the state equation of closed-loop system with actuator faults

$$\dot{x}(t) = A_{cf}x(t) \tag{7}$$

where $A_{cf} = A + BMK + E\Sigma(F_1 + F_2MK)$.

In summary, the design of robust fault-tolerant controller under region pole constraint can be described as follows: In terms of actuator fault model (6), system (7) with the parameter perturbations can obtain a controller (3), which allocates all poles of fault closed-loop system to a certain circular region $D(r,\ q)$, then the controller (3) is called as the robust D-stabilized fault-tolerant controller. Thus the solvability of controller can be transferred into that of a set of LMIs.

3 Design of Robust D -Stabilized Fault-Tolerant Controller

Before processing norm bounded perturbations and actuator faults, we need the following lemma:

Lemma 2[7]. For any matrixes X , Y with appropriate dimensions and scalar $\varepsilon > 0$, it follows that $X^T Y + Y^T X \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y$.

Lemma 3[7]. Foe any matrixes Y、D and E with appropriate dimensions, where Y is symmetrical, then for all matrix F that meet $F^T F \le I$, $Y + DFE + E^T F^T D^T < 0$ holds, if and only if there exists scalar $\varepsilon > 0$ such that $Y + \varepsilon^{-1} DD^T + \varepsilon E^T E < 0$.

Theorem 1. Given a circular region $D(r, q)$, a D-stabilized routine control law for system (1) is achievable, if and only if there exist scalar $\varepsilon_1 > 0$, matrix $P > 0$ and matrix V such that LMI

$$\begin{bmatrix} -rP + \varepsilon_1 EE^T & (A+qI)P + BV & 0 \\ [(A+qI)P + BV]^T & -rP & (F_1 P + F_2 V)^T \\ 0 & F_1 P + F_2 V & -\varepsilon_1 I \end{bmatrix} < 0 \tag{8}$$

then $(P$、V、$\varepsilon_1)$ is a feasible solution of control law as follows:

$$u^*(t) = VP^{-1}x(t) \tag{9}$$

Proof. Given a circular region $D(r, q)$, it's easy to verify that inequality (5) is equivalent to

$$\begin{bmatrix} -rP & A_c \\ A_c^T & -rP \end{bmatrix} < 0$$

Substitute A_c in above

$$\begin{bmatrix} -rP & (A+qI)P + BKP \\ [(A+qI)P + BKP]^T & -rP \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} \Sigma [0 \quad (F_1 + F_2 K)P] + [0 \quad (F_1 + F_2 K)P]^T \Sigma^T \begin{bmatrix} E \\ 0 \end{bmatrix}^T < 0$$

From lemma 3, for all uncertain matrix Σ that meet $\Sigma^T \Sigma \le I$, the above inequality holds, if and only if there exists scalar $\varepsilon_1 > 0$ such that

$$\begin{bmatrix} -rP & (A+qI)P + BKP \\ [(A+qI)P + BKP]^T & -rP \end{bmatrix} + \varepsilon_1 \begin{bmatrix} E \\ 0 \end{bmatrix} \begin{bmatrix} E^T & 0 \end{bmatrix} + \varepsilon_1^{-1} \begin{bmatrix} 0 \\ [(F_1 + F_2 K)P]^T \end{bmatrix} [0 \quad (F_1 + F_2 K)P] < 0$$

that is

$$\begin{bmatrix} -rP + \varepsilon_1 EE^T & (A+qI)P + BKP \\ [(A+qI)P + BKP]^T & -rP \end{bmatrix} - \begin{bmatrix} 0 \\ [(F_1 + F_2 K)P]^T \end{bmatrix} (-\varepsilon_1^{-1} I) [0 \quad (F_1 + F_2 K)P] < 0$$

By application of Schur complement theory, an alternative way of writing this is

$$\begin{bmatrix} -rP+\varepsilon_1 EE^T & \begin{matrix}(A+qI)P \\ +BKP\end{matrix} & 0 \\ \begin{matrix}[(A+qI)P \\ +BKP]^T\end{matrix} & -rP & (F_1P+F_2KP)^T \\ 0 & F_1P+F_2KP & -\varepsilon_1 I \end{bmatrix} < 0$$

Set $V = KP$, then completes the proof.

Theorem 2. Given a circular region $D(r,\ q)$ and the occurrence of actuator failures, a robust D-stabilized fault-tolerant control law for system (1) is achievable, if and only if there exist scalar $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and positive definite matrix P、V such that LMI with symmetrical structure

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} & 0 \\ * & -rP & H_{23} & V^T J^{1/2} \\ * & * & H_{33} & 0 \\ * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \tag{10}$$

where

$$\begin{aligned} H_{11} &= -rP+\varepsilon_1 EE^T+\varepsilon_2 BM_0JM_0B^T \\ H_{12} &= (A+qI)P+BM_0V \\ H_{13} &= \varepsilon_2 BM_0JM_0F_2^T \\ H_{23} &= (F_1P+F_2M_0V)^T \\ H_{33} &= \varepsilon_2 F_2M_0JM_0F_2^T-\varepsilon_1 I \end{aligned}$$

Proof. Consider fault matrix $M = M_0(I+L)$, the inequality (8) can be written as

$$\begin{bmatrix} -rP+\varepsilon_1 EE^T & \begin{matrix}(A+qI)P \\ +BMV\end{matrix} & 0 \\ \begin{matrix}[(A+qI)P \\ +BMV]^T\end{matrix} & -rP & (F_1P+F_2MV)^T \\ 0 & F_1P+F_2MV & -\varepsilon_1 I \end{bmatrix} < 0$$

that is

$$\begin{bmatrix} -rP+\varepsilon_1 EE^T & \begin{matrix}(A+qI)P \\ +BM_0V\end{matrix} & 0 \\ * & -rP & (F_1P+F_2M_0V)^T \\ * & * & -\varepsilon_1 I \end{bmatrix} + \begin{bmatrix} BM_0J^{1/2} \\ 0 \\ F_2M_0J^{1/2} \end{bmatrix} \begin{bmatrix} 0 & J^{1/2}V & 0 \end{bmatrix} + \left[\begin{bmatrix} BM_0J^{1/2} \\ 0 \\ F_2M_0J^{1/2} \end{bmatrix} \begin{bmatrix} 0 & J^{1/2}V & 0 \end{bmatrix} \right]^T < 0$$

According to Schur complement property and lemma 2, the above formula can be transferred into inequality (10) with further consolidation, then completes the proof.

Furthermore, given system (1) and region D, with respect to the following optimal problem

$$\min_{P,V} Trace(S)$$

such that:（i）linear matrix inequality (10) holds;（ii）$\begin{bmatrix} S & I \\ I & P \end{bmatrix} > 0$

where the constraint（ii）is equal to $S > P^{-1} > 0$. The minimum of $Trace(S)$ will ensure the minimum of $Trace(P^{-1})$, that is, the minimum allowable bound of system performance. With solution $(P、V)$, the formula (9) is called as the optimal robust fault-tolerant control law of the system.

Remark. According to (10), the fault-tolerant controller design problem of the system is equivalent to determine the optimum solutions $(P、V、\varepsilon_1、\varepsilon_2)$ of LMI, by utilizing LMI Control Toolbox which implements interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [10][11].

4 Simulation Example

Consider the operating condition of X type of turbofan engine in a state of $H = 0km$ 、 $Ma = 0$, the double variant normalized linear model is built with parameter matrixes

$$A = \begin{bmatrix} -2.3642 & -0.3014 \\ 3.0958 & -2.8747 \end{bmatrix}, \quad B = \begin{bmatrix} 0.6978 & 0.9386 \\ 0.6834 & 1.7602 \end{bmatrix}$$

the uncertain decomposition matrix as

$$E = \begin{bmatrix} 0.236 & 0.381 \\ 0 & 0.252 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 0.6 \end{bmatrix}$$

and with the nominal system poles $-2.6194 \pm j0.9316$.

Given the dynamic performance index: attenuation $\alpha \geq 3$, overshoot $\sigma\% \leq 5\%$, accommodation time $t_s \leq 4.6s$, the pole of closed-loop system can be assigned into the circular region $D(2,\ 5)$.

According to theorem 1, the robust D-stabilized routine control law $u^*(t) = K_1 x(t)$ can be obtained by the solution of LMI (8) where the variable is $(P,\ V)$. One state feedback gain matrix is

$$K_1 = VP^{-1} = \begin{bmatrix} -2.8806 & 3.9930 \\ -0.6422 & -2.7353 \end{bmatrix}$$

In the case of fuel controlling, such throttle switch and fuel distribution device closely related to oil-supplied characteristics are vulnerable to oil leakage caused by assembly quality or sealing failure. On the other hand, hydraulic fluid valve contaminated by work pollutants will easily lead to jet actuator stuck tightly. Here lord fuel metering valve and jet actuator fault are simulated respectively as fashions $m_1 \in [0.8,1]$ and $m_2 \in [0.4,1]$. Work out

$$M_0 = diag\{0.9,\ 0.7\},\quad J = diag\{0.1111,\ 0.4286\}$$

According to theorem 2, calculate the robust D-stabilized fault-tolerant control law $u^*(t) = K_2 x(t)$, where the state feedback gain matrix is

$$K_2 = VP^{-1} = \begin{bmatrix} -1.6814 & -0.3413 \\ -1.4738 & -0.5560 \end{bmatrix}$$

4.1 Robust Stability Analysis

Take into account 10 groups of model with structural parameter norm bounded perturbations $\|\Delta A\| \le 0.4$, $\|\Delta B\| \le 0.3$. Given that

(1) $M = I$, i.e. actuators in normal condition;

(2) $M = diag\{1,\ 0.4\}$, i.e. lord fuel metering valve works normally, but jet actuator totally stuck.

The poles of closed-loop system corresponding to two control laws distribution are shown as Fig. 2 and Fig.3. Note that closed-loop poles utilized by both control laws are all in the designated circular region under normal state. While on occurrence of actuator faults, partial poles by routine control law deviate from the designated circular region to positive complex plane, which may lead to system unstable. Yet by adopting robust fault-tolerant control law, all poles are still remain in the designated circular region, ensuring the system stability.

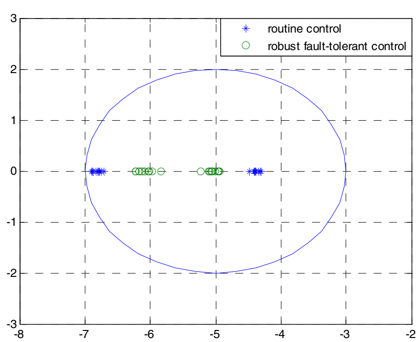

Fig. 2. Distribution of closed loop poles under condition (1)

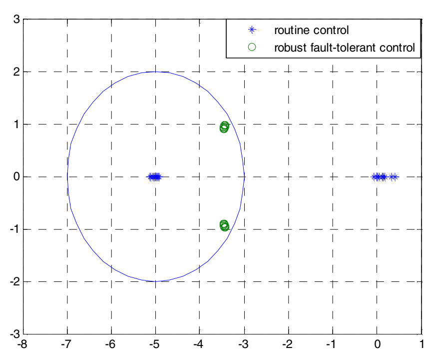

Fig. 3. Distribution of closed loop poles under condition (2)

4.2 Robust Fault-Tolerant Performance Analysis

Consider the actuator faults at $t = 5s$,

(1) $M = diag\{1,\ 0.6\}$, i.e. jet actuator totally stuck faults;

(2) $M = diag\{0.8,\ 1\}$, i.e. lord fuel metering valve leak faults.

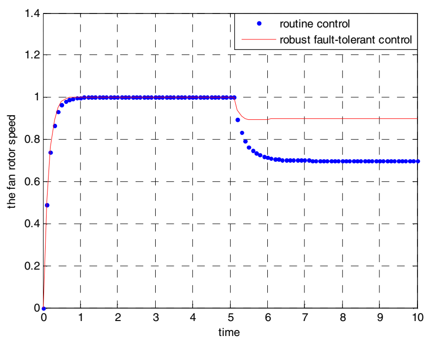

Fig. 4. Step response of the fan rotor speed under fault (1)

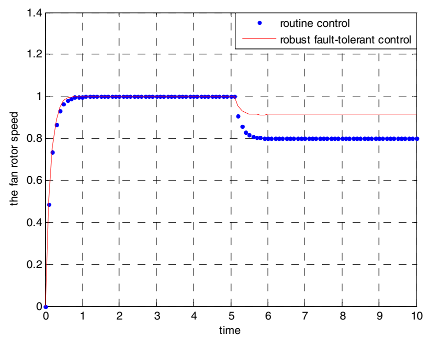

Fig. 5. Step response of the fan rotor speed under fault (2)

By Application of two control laws, the step response cure of the fan rotor speed n_L under the assumption of above faults are shown as Fig.4 and Fig.5. Note that the robust fault-tolerant control law proposed is obviously superior to routine control law. With strong fault-tolerant capability for both actuator faults, the dynamic performance remains approximate adjust to normal state.

5 Conclusion

A general form of turbine line dynamic model with norm bounded perturbations is in discussion. By adoption of robust fault-tolerant control approach based on LMI, a state feedback fault tolerant controller is designed to satisfy the regional poles constraints and performance in actuator fault condition and structured perturbations. Simulation results illustrate that, the proposed approach possesses strong fault-tolerant capability and maintains the performance of robust stability in designated circular region. Furthermore, this approach can be expanded to sensor faults situation.

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