Angular Rate Constrained Attitude Reorientation of Rigid body

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***Abstract*— This paper presents a solution for rest-to-rest attitude reorientation for a rigid body under angular velocity constraints and external disturbances. Assuming that external disturbances are unknown but bounded, the sliding mode technique is used in attitude controller to reject disturbances. Then, a potential function in terms of sliding vector is proposed with a largest potential placing at the maximal angular rate. Based on the potential function, an adaptive sliding mode controller is developed to stabilize the closed-loop attitude control system. Finally, the efficiency of the proposed attitude control scheme is demonstrated by numerical simulations.**

I. INTRODUCTION

Rigid-body attitude control is one of the most widely studied research field in control literature with rich technically results dealing with multiple application-specific constraints. For rigid spacecraft implementations, due to the saturation limitation of low-rate gyro or mission specification requirement, angular velocity constraint needs to be taken into account for attitude control. An practical example is X-Ray Timing Explorer (XTE) spacecraft that is required to maneuver within the saturation limit of rate gyros [1]. In view of these practical considerations, this paper investigates the reorientation problem for a rigid body subjecting to angular velocity constraints.

A satellite's motion is governed by kinematic and dynamic equations, and the mathematical models are highly nonlinear and coupled. Extensive nonlinear control algorithms have been proposed for solving the spacecraft reorientation problem, such as proportional-derivative feedback control [2], [3], sliding mode control [4], [5], [6], backstepping control [7], [8], adaptive control [9], [10], and inverse optimal control [11], [12]. However, it should be noted that angular velocity constraint is not taken into account in above mentioned literatures. To ensure that angular velocity is always within a predefined bound determined by saturation limit of rate gyros or performance requirements, several methods has been proposed. In [1], a cascade-saturation attitude control logic was developed for the near-minimum-time eigenaxis reorientation problem of the XTE spacecraft when angular velocity and control torque constraints are taken into account. Although this approach has been widely adopted in practical attitude control systems, a rigorous stability proof of the closed-loop system was not given. In [16], a time-efficient angular steering law was presented to accommodate a variety of system limitations including angular rate constraints, where a braking curve is designed to determine the angular rate and acceleration limits. In [17], a robust nonlinear feedback control strategy incorporating with control allocation scheme was proposed to achieve attitude stabilization under designed angular velocity constraints and actuator saturation, where logarithmic barrier potential function in terms of angular velocity is used in Lyapunov function to analyze system stability despite the angular velocity constraints. In [18], an attitude stabilization algorithm accounting for angular velocity constraints and unwinding phenomenon was designed to ensure a fast and accurate response for a rigid spacecraft.

In this paper, based on potential function defined in angular velocity domain, an adaptive attitude controller is presented for solving rigid-body reorientation problem with consideration of angular rate limits and external disturbances. The attitude of a rigid body is parameterized by unit quaternion, which can represent attitude globally. A potential function in terms of sliding vector is proposed so that angular velocity constraint is ensured by limiting the magnitude of the sliding vector. Consequently, assuming that the external disturbances are bounded, an adaptive sliding mode controller based on the proposed potential functions is proposed to achieve uniform ultimate boundedness of the closed-loop system.

The remainder of this paper is organized as follows. In Section II, unit-quaternion is introduced for attitude representation, and rigid-body dynamics and modelling of angular rate limits are described. In Section III, a logarithmical potential function, are designed to describe angular velocity limits. Then, an adaptive attitude control law using sliding mode control technique is developed to guarantee that the closed-loop system is uniformly ultimately bounded stable. The simulation results are given in Section IV, followed by conclusions in Section V.

II. MATHEMATICAL MODELS

In this paper, the unit-quaternion representation is used to describe the orientation of a rigid body. A quaternion is defined as $\boldsymbol{Q} = [q_1 \ q_2 \ q_3 \ q_0]^T = [\boldsymbol{q}^T \ q_0]^T \in \boldsymbol{\mathcal{Q}}$, where the vector part $\boldsymbol{q} \in \boldsymbol{\mathcal{R}}^3$, the scalar part $q_0 \in \boldsymbol{\mathcal{R}}$, and $\boldsymbol{\mathcal{Q}}$ denotes the set of quaternion. The notation "$\otimes$" denotes the quaternion multiplication operator of two quaternion $\boldsymbol{Q}_i = [\boldsymbol{q}_i^T \ q_{i0}]^T \in \boldsymbol{\mathcal{Q}}$ and $\boldsymbol{Q}_j = [\boldsymbol{q}_j^T \ q_{j0}]^T \in \boldsymbol{\mathcal{Q}}$, and is defined as

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follows:

$$\boldsymbol{Q}_i \otimes \boldsymbol{Q}_j = \begin{bmatrix} q_{i0}\boldsymbol{q}_j + q_{j0}\boldsymbol{q}_i + \boldsymbol{S}(\boldsymbol{q}_i)\boldsymbol{q}_j \\ q_{i0}q_{j0} - \boldsymbol{q}_i^T\boldsymbol{q}_j \end{bmatrix}, \tag{1}$$

and has the quaternion $\boldsymbol{Q}_{\boldsymbol{I}} = [0\ 0\ 0\ 1]^T$ as identity element. The matrix $\boldsymbol{S}(\boldsymbol{x}) \in \boldsymbol{\mathcal{R}}^{\boldsymbol{3\times 3}}$ is a skew-symmetric matrix satisfying $\boldsymbol{S}(\boldsymbol{x})\boldsymbol{y} = \boldsymbol{x} \times \boldsymbol{y}$ for any vectors $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{\mathcal{R}}^{\boldsymbol{3}}$, and $\times$ denotes vector cross product. The set of unit quaternion $\boldsymbol{\mathcal{Q}}_u$ is a subset of quaternion $\boldsymbol{\mathcal{Q}}$ such that

$$\boldsymbol{\mathcal{Q}}_u = \{\boldsymbol{Q} = [\boldsymbol{q}^T\ q_0]^T \in \boldsymbol{\mathcal{R}}^3 \times \boldsymbol{\mathcal{R}} \mid \boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1\}, \tag{2}$$

where the vector part $\boldsymbol{q} = \hat{\boldsymbol{n}}\sin(\frac{\phi}{2})$, and the scalar part $q_0 = \cos(\frac{\phi}{2})$; $\hat{\boldsymbol{n}}$ and ϕ refer to the Euler axis and the rotation angle about the Euler axis. The unit-quaternion conjugate or inverse is defined as $\boldsymbol{Q}^* = [-\boldsymbol{q}^T\ q_0]^T$. The quaternion satisfies the following important properties [13]:

$$\boldsymbol{\alpha} \otimes (\boldsymbol{\beta} \pm \boldsymbol{\gamma}) = \boldsymbol{\alpha} \otimes \boldsymbol{\beta} \pm \boldsymbol{\alpha} \otimes \boldsymbol{\gamma} \tag{3}$$
$$(\boldsymbol{\alpha} \otimes \boldsymbol{b})^* = \boldsymbol{\beta}^* \otimes \boldsymbol{\alpha}^* \tag{4}$$
$$(\varrho\boldsymbol{\alpha}) \otimes \boldsymbol{b} = \boldsymbol{\alpha} \otimes (\varrho\boldsymbol{\beta}) = \varrho(\boldsymbol{\alpha} \otimes \boldsymbol{\beta}) \tag{5}$$
$$(\boldsymbol{\alpha} \otimes \boldsymbol{\beta}) \otimes \boldsymbol{\gamma} = \boldsymbol{\alpha} \otimes (\boldsymbol{\beta} \otimes \boldsymbol{\gamma}) \tag{6}$$
$$\boldsymbol{\alpha}^T(\boldsymbol{\beta} \otimes \boldsymbol{\gamma}) = \boldsymbol{\gamma}^T(\boldsymbol{\beta}^* \otimes \boldsymbol{\alpha}) = \boldsymbol{\beta}^T(\boldsymbol{\alpha} \otimes \boldsymbol{\gamma}^*), \tag{7}$$

where ϱ is a constant, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ are quaternions belonging to the set $\boldsymbol{\mathcal{Q}}$.

A. Kinematics equation

The spacecraft kinematics in terms of unit-quaternion can be given by

$$\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{Q} \otimes \boldsymbol{\nu}(\boldsymbol{\omega}) = \frac{1}{2}\begin{bmatrix} \boldsymbol{S}(\boldsymbol{q}) + q_0\boldsymbol{I}_3 \\ -\boldsymbol{q}^T \end{bmatrix}\boldsymbol{\omega}, \tag{8}$$

where $\boldsymbol{Q} = [q_1\ q_2\ q_3\ q_0]^T = [\boldsymbol{q}^T\ q_0]^T \in \boldsymbol{\mathcal{Q}}_u$ denotes the unit-quaternion describing the attitude orientation of the body frame $\boldsymbol{\mathcal{B}}$ with respect to inertial frame $\boldsymbol{\mathcal{I}}$ and satisfies the constraint $\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1$, $\boldsymbol{\omega} \in \boldsymbol{\mathcal{R}}^{\boldsymbol{3}}$ is the inertial angular velocity vector of the spacecraft with respect to an inertial frame $\boldsymbol{\mathcal{I}}$ and expressed in the body frame $\boldsymbol{\mathcal{B}}$, and the function $\boldsymbol{\nu}$: $\boldsymbol{\mathcal{R}}^3 \to \boldsymbol{\mathcal{R}}^4$ is defined as the mapping $\boldsymbol{\nu}(\boldsymbol{\omega}) = [\boldsymbol{\omega}^T\ 0]^T$.

Let $\boldsymbol{Q}_d \in \boldsymbol{\mathcal{Q}}_u$ denote the desired attitude. In this paper, the rest-to-rest attitude reorientation problem of rotating a rigid body from its current attitude $\boldsymbol{Q}$ to a desired attitude $\boldsymbol{Q}_d$ is considered. The unit-quaternion error $\boldsymbol{Q}_e = [q_{e1}\ \ q_{e2}\ \ q_{e3}\ \ q_{e0}]^T = [\boldsymbol{q}_e^T\ q_{e0}]^T \in \boldsymbol{\mathcal{Q}}_u$ is defined as $\boldsymbol{Q}_e = \boldsymbol{Q}_d^* \otimes \boldsymbol{Q} = [\boldsymbol{q}_e^T\ q_{e0}]^T$, which describes the discrepancy between the actual unit-quaternion $\boldsymbol{Q}$ and the desired unit-quaternion $\boldsymbol{Q}_d$. The kinematics represented by unit-quaternion error is described as [14]

$$\dot{\boldsymbol{Q}}_e = \frac{1}{2}\boldsymbol{Q}_e \otimes \boldsymbol{\nu}(\boldsymbol{\omega}_e), \tag{9}$$

where $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{R}(\boldsymbol{Q}_e)^T\boldsymbol{\omega}_d$, $\boldsymbol{R}(\boldsymbol{Q}_e)$ is the unit-quaternion error $\boldsymbol{Q}_e$ related rotation matrix [14] defined as $\boldsymbol{R}(\boldsymbol{Q}_e) = (q_{e0}^2 - \boldsymbol{q}_e^T\boldsymbol{q}_e)\boldsymbol{I}_3 + 2\boldsymbol{q}_e\boldsymbol{q}_e^T - 2q_{e0}\boldsymbol{S}(\boldsymbol{q}_e)$, and $\boldsymbol{\omega}_d$ denotes the desired angular velocity. In this paper, since only rest-to-rest attitude reorientation problem is considered, the desired angle velocity is $\boldsymbol{\omega}_d = 0$, which yields $\boldsymbol{\omega}_e = \boldsymbol{\omega}$. Therefore, the attitude error kinematics for a rest-to-rest attitude reorientation maneuver in (9) can be rewritten as

$$\dot{\boldsymbol{Q}}_e = \frac{1}{2}\boldsymbol{Q}_e \otimes \boldsymbol{\nu}(\boldsymbol{\omega}) = \frac{1}{2}\begin{bmatrix} \boldsymbol{S}(\boldsymbol{q}_e) + q_{e0}\boldsymbol{I}_3 \\ -\boldsymbol{q}_e^T \end{bmatrix}\boldsymbol{\omega}. \tag{10}$$

B. Spacecraft Dynamics

The dynamics for the attitude motion of a rigid body can be expressed by the following equation [9]:

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\tau} + \boldsymbol{d} \tag{11}$$

where $\boldsymbol{J} = \mathrm{diag}\{J_1, J_2, J_3\} \in \boldsymbol{\mathcal{R}}^{3\times 3}$ denotes the positive definite inertia matrix of a rigid body, $\boldsymbol{\tau} \in \boldsymbol{\mathcal{R}}^3$ denotes the control torque about the body axes, $\boldsymbol{d} \in \boldsymbol{\mathcal{R}}^3$ denotes the external disturbances.

To design the attitude controller, a sliding vector $\boldsymbol{s} = [s_1, s_2, s_3]^T \in \boldsymbol{\mathcal{R}}^3$ is given by

$$\boldsymbol{s} = \boldsymbol{\omega} + k\boldsymbol{q}_e, \tag{12}$$

where k is a positive constant. Consequently, the attitude dynamics in terms of the sliding vector can be written as

$$\boldsymbol{J}\dot{\boldsymbol{s}} = \boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e) + \boldsymbol{\tau} + \boldsymbol{d}, \tag{13}$$

where the nonlinear term $\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e) = -\boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{J}\boldsymbol{\omega} + \frac{k}{2}\left(\boldsymbol{S}(\boldsymbol{q}_e) + q_{e0}\boldsymbol{I}_3\right)\boldsymbol{\omega}$.

Assumption 1: The external disturbance $\boldsymbol{d}$ is bounded such that $\|\boldsymbol{d}\| \le d_{\max}$, where $d_{\max}$ is a positive constant and $\|\cdot\|$ denotes the Euclidean norm.

C. Angular Velocity Constraints

Due to the limited measurement range of the rate gyros or specific mission requirements, the constraints on angular velocity might be required. Suppose that the angular velocity information is available, the set of angular velocity constraint is represented as

$$\boldsymbol{\mathcal{W}} = \left\{\boldsymbol{\omega} \in \mathcal{R}^3 \mid |\omega_i| \le \omega_{i,\max}\right\} \tag{14}$$

where $\omega_{i,\max}$ $(i = 1, 2, 3)$ is the limitation of allowable operational angular velocities for each axis.

III. ADAPTIVE ATTITUDE REORIENTATION CONTROLLER DESIGN

A. Potential Function for Angular Velocity Constraints

In addition, to satisfy the angular velocity constraints, a logarithmic potential function $V_r(\boldsymbol{s})$: $\boldsymbol{\mathcal{S}}_r \to \boldsymbol{\mathcal{R}}$, is proposed as

$$\begin{aligned} V_r(\boldsymbol{s}) =& \frac{1}{2}\left[\log\left(\frac{s_{1,\max}^2}{s_{1,\max}^2 - s_1^2}\right) + \log\left(\frac{s_{2,\max}^2}{s_{2,\max}^2 - s_2^2}\right)\right. \\ &\left. + \log\left(\frac{s_{3,\max}^2}{s_{3,\max}^2 - s_3^2}\right)\right] \\ =& \frac{1}{2}\sum_{i=1}^{3}\log\left(\frac{s_{i,\max}^2}{s_{i,\max}^2 - s_i^2}\right), \end{aligned} \tag{15}$$

where the sliding vector permissible zone $\boldsymbol{\mathcal{S}}_r$ is specified as

$$\boldsymbol{\mathcal{S}}_r = \left\{ \boldsymbol{s} \in \boldsymbol{\mathcal{R}}^3 \mid |s_i| \le s_{i,\max} \right\} \tag{16}$$

and $s_{i,\max}$ is a pre-defined maximal constant value satisfying $s_{i,\max} = \omega_{i,\max} - k > 0$ for s_i $(i = 1, 2, 3)$. Meanwhile, it is assumed that $\boldsymbol{s}(0) \in \boldsymbol{\mathcal{S}}_r$.

The above logarithmic potential function guarantees that the angular velocity always stays in constrained zone defined in (14).

Lemma 1: The potential function in (15) has the following properties:

1) $V_r(0) = 0$
2) $V_r(\boldsymbol{s}) > 0$, for all $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r \backslash \{0\}$
3) If $s_{i,\max} = \omega_{i,\max} - k > 0$ for all $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r$, then $|\omega_i| < \omega_{i,max}$
4) $\nabla^2 V_r(\boldsymbol{s}) > 0$ is positive definite for all $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r$.

Proof: From the definition of the logarithmic potential function $V_r(\boldsymbol{s})$, it is clear that $V_r(0) = 0$. Moreover, for all $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r \backslash \{0\}$, the inequalities

$$\frac{s_{i,\max}^2}{s_{i,\max}^2 - s_i^2} > 1 \tag{17}$$

hold, which subsequently leads to

$$\log \left(\frac{s_{i,\max}^2}{s_{i,\max}^2 - s_i^2} \right) > 0. \tag{18}$$

Hence, $V_r(\boldsymbol{s}) > 0$, for all $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r \backslash \{0\}$.

The third part of ***Lemma 1*** guarantees that the angular velocity always stays in constrained zone defined in (14) when the maximal value for sliding vector is chosen as $s_{i,\max} = \omega_{i,\max} - k > 0$. On the one hand, since $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r$ ensures $|s_i| \le s_{i,\max}$, it is found that $\omega_{i,\max} - k \ge |s_i|$. On the other hand, in view of sliding vector in (12) and the unit-quaternion property that $|q_{ei}| \le 1$, it is clear that $|s_i| \ge |\omega_i| - k$. Therefore, in view of these two aspects, it can be obtained from the potential function (15) that $|\omega_i| \le \omega_{i,\max}$.

The last part of ***Lemma 1*** can be shown by taking the second order partial derivative of $V_r(\boldsymbol{s})$ with respect to $\boldsymbol{s}$. Since the potential function $V_r(\boldsymbol{s})$ is a linear combination of three logarithmic functions, it is sufficient to analyze one of the terms in more details, for example,

$$V_{ri}(s_i) = \frac{1}{2} \log \left(\frac{s_{i,\max}^2}{s_{i,\max}^2 - s_i^2} \right). \tag{19}$$

The gradient of $V_{ri}(s_i)$ is calculated as

$$\nabla V_{ri}(s_i) = \frac{s_i}{s_{i,\max}^2 - s_i^2}. \tag{20}$$

Consequently, the Hessian $\nabla^2 V_{ri}(s_i)$ is given as

$$\nabla^2 V_{ri}(s_i) = \frac{s_{i,\max}^2 + s_i^2}{(s_{i,\max}^2 - s_i^2)^2}, \tag{21}$$

which implies that $\nabla^2 V_r(s_i) > 0$. Therefore, it is clear that $\nabla^2 V_r(\boldsymbol{s_i}) > 0$ if $\boldsymbol{s} \in \boldsymbol{\mathcal{S}}_r$. ■

B. ADAPTIVE CONTROLLER DESIGN

The adaptive attitude reorientation controller is designed as

$$\boldsymbol{\tau} = -\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e) - k_1 \boldsymbol{\Upsilon} \boldsymbol{s} - \hat{\bar{d}} \frac{\boldsymbol{\Upsilon}^{-1} \boldsymbol{s}}{\|\boldsymbol{\Upsilon}^{-1} \boldsymbol{s}\|} \tag{22}$$

with

$$\dot{\hat{\bar{d}}} = \rho \left(\|\boldsymbol{\Upsilon}^{-1} \boldsymbol{s}\| - \mu \hat{\bar{d}} \right), \tag{23}$$

where the operator Vec$[\cdot]$ denotes the vector part of $[\cdot]$, $\boldsymbol{\Upsilon} = \text{diag}\left\{ J_1(s_{1,\max}^2 - s_1^2), J_2(s_{2,\max}^2 - s_2^2), J_3(s_{3,\max}^2 - s_3^2) \right\}$, the variables k_1, ρ, and μ are positive constants.

Consider the following Lyapunov candidate:

$$\begin{aligned} V_\ell =& 2kk_1[\boldsymbol{q}_e^T \boldsymbol{q}_e + (1 - q_0)^2] + V_r(\boldsymbol{s}) \\ &+ \frac{1}{2\rho} (\hat{\bar{d}} - d_{\max})^2. \end{aligned} \tag{24}$$

The time derivative of V_ℓ is

$$\begin{aligned} \dot{V}_\ell =& 2kk_1 \boldsymbol{q}_e^T \boldsymbol{\omega} + \boldsymbol{s}^T \boldsymbol{\Upsilon}^{-1} \boldsymbol{J} \dot{s} + \frac{1}{\rho} (\hat{\bar{d}} - d_{\max}) \dot{\hat{\bar{d}}} \\ =& 2kk_1 \boldsymbol{\omega}^T \boldsymbol{q}_e + \boldsymbol{s}^T \boldsymbol{\Upsilon}^{-1} \left[\boldsymbol{f}(\boldsymbol{\omega}, \boldsymbol{Q}_e) + \boldsymbol{\tau} + \boldsymbol{d} \right] \\ &+ \frac{1}{\rho} (\hat{\bar{d}} - d_{\max}) \dot{\hat{\bar{d}}}. \end{aligned} \tag{25}$$

Then, substituting the controller (22) and adaptive laws (23) in above equation, it yields

$$\begin{aligned} \dot{V}_\ell =& 2kk_1 \boldsymbol{\omega}^T \boldsymbol{q_e} - k_1 \boldsymbol{s}^T \boldsymbol{s} - \hat{\bar{d}} \|\boldsymbol{\Upsilon}^{-1} \boldsymbol{s}\| + \boldsymbol{s}^T \boldsymbol{\Upsilon}^{-1} \boldsymbol{d} \\ &+ (\hat{\bar{d}} - d_{\max}) \left(\|\boldsymbol{\Upsilon}^{-1} \boldsymbol{s}\| - \mu \hat{\bar{d}} \right) \\ \le& - k_1 k^2 \boldsymbol{q}_e^T \boldsymbol{q}_e - k_1 \boldsymbol{\omega}^T \boldsymbol{\omega} - \mu \hat{\bar{d}} (\hat{\bar{d}} - d_{max}) \\ \le& - k_1 k^2 \|\boldsymbol{q_e}\|^2 - k_1 \|\boldsymbol{\omega}\|^2 - \mu (\hat{\bar{d}} - \frac{\hat{d}_{max}}{2})^2 + \frac{\mu}{4} d_{max}^2 \\ \le& - k_1 k^2 \|\boldsymbol{q_e}\|^2 - k_1 \|\boldsymbol{\omega}\|^2 + \frac{\mu}{4} d_{max}^2 \end{aligned} \tag{26}$$

From inequality (26), it can be shown that the closed-loop systems is uniformly ultimately bounded stable [15], and the signals $\boldsymbol{q_e}$ and $\boldsymbol{\omega}$ are all bounded for all $t \ge 0$. Furthermore, according to (26), it is clear that $\dot{V}_\ell < 0$ if

$$\|\boldsymbol{q}_e\| > \frac{d_{max}}{2k} \sqrt{\frac{\mu}{k_1}}, \quad \texttt{or} \quad \|\boldsymbol{\omega}\| > \frac{d_{max}}{2} \sqrt{\frac{\mu}{k_1}} \tag{27}$$

As a result, the decrease of $\dot{V}_\ell$ drives $\|\boldsymbol{q}_e\|$ and $\|\boldsymbol{\omega}_a\|$ into $\|\boldsymbol{q}_e\| \le \frac{d_{max}}{2k} \sqrt{\frac{\mu}{k_1}}$ and $\|\boldsymbol{\omega}\| \le \frac{d_{max}}{2} \sqrt{\frac{\mu}{k_1}}$, respectively.

In summary, we have the following theorem.

Theorem 1: The attitude controller (22) with adaptive laws (23), applied to rigid-body attitude kinematics and dynamics expressed by (8) and (11), guarantees that all closed-loop signals are bounded and attitude error and angular velocity converge to a compact set containing zero despite the angular rate limits and external disturbances.

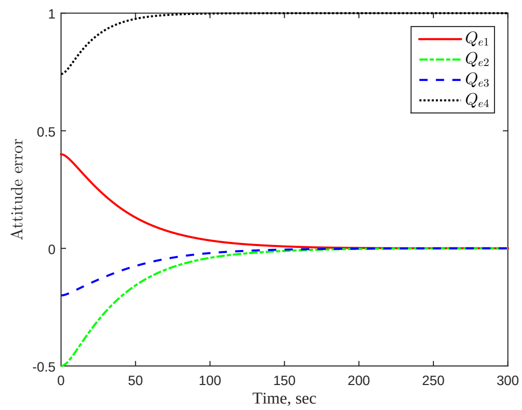

Fig. 1. Time history of attitude.

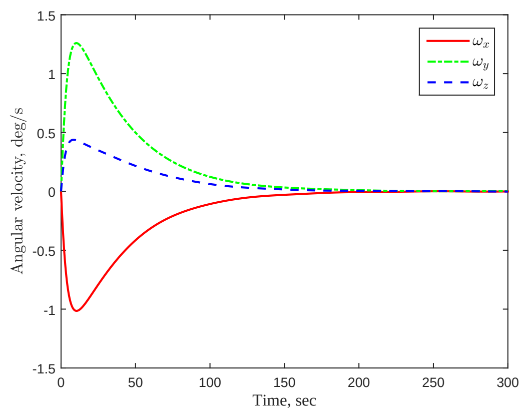

Fig. 2. Time history of angular velocity.

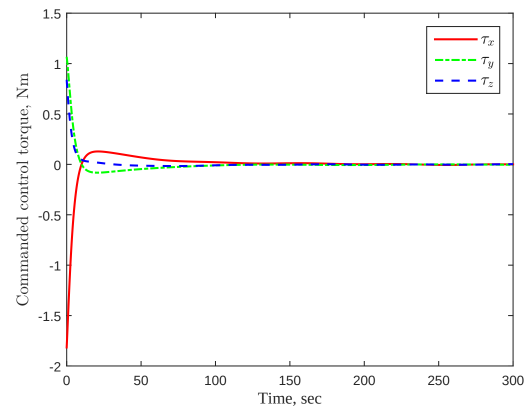

Fig. 3. Time history of commanded control torque

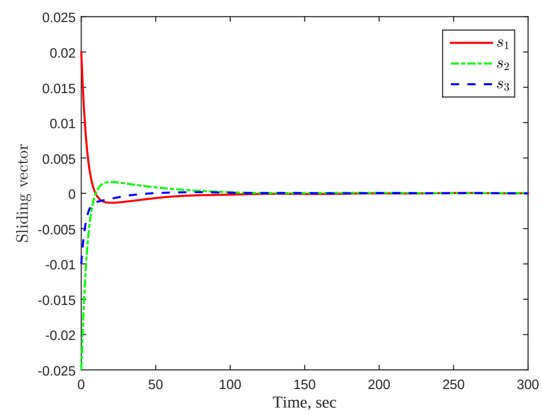

Fig. 4. Sliding vector

IV. SIMULATION RESULTS

To demonstrate the effectiveness and performance of the proposed controller, numerical simulation is carried out to a rigid spacecraft in this section. The inertia matrix of spacecraft is $\boldsymbol{J} = \mathrm{diag}([350, 180, 290])$ kg $\cdot$ m^2. The spacecraft is assumed to have the initial attitude $\boldsymbol{Q}(0) = [0.33\ 0.66\ \ -0.62\ \ -0.2726]^T$ and initial angular velocity $\boldsymbol{\omega}(0) = [0\ \ 0\ \ 0]^T$ deg/s. The desired attitude is $\boldsymbol{Q}_d = [0.2\ \ -0.55\ \ -0.42\ \ -0.5027]^T$. The angular rate limit for each axis is set to be 6 deg/s. The control gains in (22) are chosen as $k = 0.05$, $k_1 = 0.364\boldsymbol{J}$, $\rho = 0.01$, $\mu = 0.01$, and $\hat{\bar{d}}(0) = 0.001$.

The simulation results are shown in Figures 1 to 4. Figures 1 and 2 depict the attitude and angular velocity responses of spacecraft controller by the proposed method. It is observed that attitude and angular velocity converge to zero eventually. Moreover, the angular velocity is always less than its allowable maximum. The commanded control torque from attitude controller is shown in Figure 3. The response of sliding vector is shown Figure 4, from which it is clear that the magnitude of sliding vector is also limited.

V. CONCLUSIONS

This paper addresses the problem of constrained attitude controller design for a rigid body in the presence of angular rate limits and external disturbances. To ensure that the specified maximum angular rate is not exceeded, a logarithmic potential function is developed in terms of sliding vector. Consequently, an adaptive attitude control law is synthesized to achieve uniformly ultimately stability of the overall closed-loop attitude control system. Numerical simulation examples are provided to show the efficiency of the proposed method.

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