Spacecraft Reorientation Control with Attitude and Velocity Constrains

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***Abstract*—This paper investigates rest-to-rest attitude reorientation problem for a rigid spacecraft in the presence of attitude and angular velocity constraints. Based on a potential function, a nonlinear attitude controller is developed to avoid the undesired celestial objects autonomously and limit the spacecraft rotation speed while achieving asymptotic attitude stabilization. To improve the agility of spacecraft, control moment gyro (CMG) is considered as torque-generating actuators for attitude control. General singular robust (GSR) steering logic is employed to determine the CMG gimbal rate commands. The proposed attitude control scheme has simple structure, which is of great interest for aerospace industry when onboard computing power is limited. Finally, simulation results for a CMG-based spacecraft are presented to show the effectiveness of the proposed attitude control systems.**

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1. INTRODUCTION

Attitude control problem for a spacecraft has been widely studied over the last decades. It plays an important role in accomplishing spacecraft missions, such as remote sensing, imaging and antenna communication. As the equations that govern attitude kinematics and dynamics are nonlinear and highly coupled, it increases difficulties in attitude controller design. Several interesting solutions to the attitude control problem have been proposed in the literature (see, for instance, [1–4]).

In practical spacecraft systems, one of their essential functions is to point an on-board instrument's boresight along a prescribed inertial direction [5]. In such a mission, instruments equipped on the spacecraft are required to be kept sufficiently far away from unwanted celestial objects or bright source of energy. In view of this requirement, the capacity of attitude controller to handle attitude constraints should be guaranteed. Otherwise, it will lead to damage of certain payloads. For example, the infrared telescopes may be required to slew from one direction in space to another without direct exposure to the sun vector or other infrared bright regions [6–8]. Generally, this type of attitude maneuver can be regarded as a spacecraft reorientation problem in the presence of attitude-constrained zones and has attracted attention in practical spacecraft missions. Potential function method formulates the attitude constrained zones in the context of an artificial potential, which is further used for synthesizing the corresponding attitude control law. It is analytical without the need of any change in the overall structure of the attitude control software or hardware,which makes it suitable for on-board computation and provides flexible autonomous operations [9].

In this paper, a potential function based attitude controller is proposed for agile spacecraft to achieve rest-to-rest attitude slew while avoiding the attitude constrained zones and satisfying angular velocity constraints. Mathematic models for spacecraft using CMGs as actuators are developed. To escape CMG singularity, general singular robust steering logic is employed. Then, unit-quaternion is used to represented the attitude constraint, and attitude-constrained zones are formulated. Consequently, potential functions are used to derive attitude controller so that attitude and angular velocity constraints are satisfied. Finally, numerical simulation using the Kent Ridge 1 satellite as model is carried out to show the effectiveness of the proposed attitude control system.

2. SPACECRAFT MATHEMATIC MODEL

In this paper, the unit-quaternion representation is used to describe the orientation of a spacecraft. A simple block diagram representation of a CMGs-based attitude control system is illustrated in Figure 1.

Dynamics Equation with CMGs

When CMGs are used as actuator for attitude control, the total angular momentum is made up of the spacecraft main body angular momentum and the actuator angular momentum, which can be expressed in the body fixed frame as follows

$$\boldsymbol{H} = \boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{A}\boldsymbol{h}, \tag{1}$$

where $\boldsymbol{J}$ is the inertia tensor, $\boldsymbol{\omega}$ is the inertial angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, $\boldsymbol{A}$ is the transformation matrix from the wheel frame to spacecraft body frame, $\boldsymbol{h}$ is the angular momentum produced by CMGs cluster. The equations of motion are derived by taking the time derivative of the total angular momentum of the system. The time derivative of $\boldsymbol{H}$ in the body frame is

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} = \boldsymbol{\tau} \tag{2}$$

$$\boldsymbol{\tau} = -(\dot{\boldsymbol{A}}\boldsymbol{h} + \boldsymbol{A}\dot{\boldsymbol{h}}) - \boldsymbol{\omega}^{\times}\boldsymbol{A}\boldsymbol{h} \tag{3}$$

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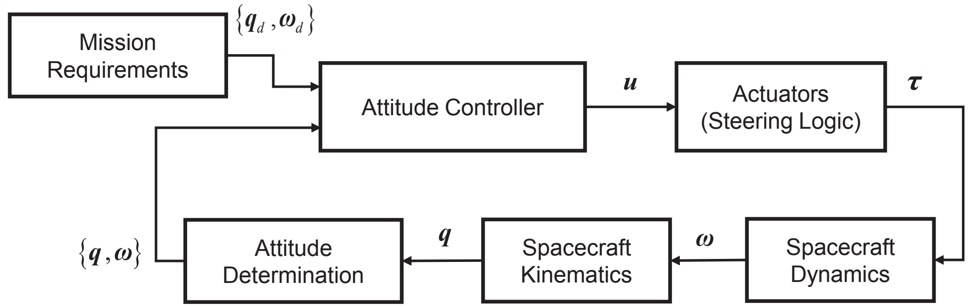

Figure 1. A CMGs-based attitude control system.

where $\boldsymbol{\tau}$ is the internal control torque generated by CMGs. The notation $\boldsymbol{a}^\times$ for a vector $\boldsymbol{a} = [a_1 \quad a_2 \quad a_3]^T$ is used to represent the skew-symmetric matrix

$$\boldsymbol{a}^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{4}$$

A CMG contains a spinning rotor with large angular momentum, but whose angular momentum vector (direction) can be changed with respect to the spacecraft by gimballing the spinning rotor. A typical single gimbal control moment gyro (SGCMG) is shown in Figure 2, in which the rotor spins at a constant speed. The angular moment vector $\boldsymbol{h}_i$ points along the spindle axis, the gimbal axis is always orthogonal to the spin axis and is denoted by $\boldsymbol{g}_i$, the output torque axis $\boldsymbol{t}_i = \boldsymbol{g}_i \times \boldsymbol{h}_i$ is orthogonal to both $\boldsymbol{g}_i$ and $\boldsymbol{h}_i$. The subscript i denotes the ith SGCMG. Vectors $\boldsymbol{g}_i$, $\boldsymbol{h}_i$ and $\boldsymbol{t}_i$ form the right hand orthogonal CMG frame and they are the unit vector in their direction. The CMG is a torque amplification device as a small gimbal torque input produces a large control torque output on the spacecraft. Because CMGs are capable of generating large control torques and angular momentum, they are often favored for precision pointing and tracking control of agile spacecraft in low Earth orbit.

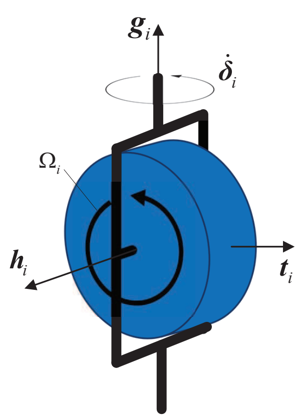

Figure 2. Single gimbal control moment gyro.

For three-axis attitude control of spacecraft, four SGCMGs in a pyramid configuration are usually selected, and the skew angle is chosen as $\beta = 54.73$ deg so that the momentum envelope is nearly spherical. Assuming that the angular momentum vector of each SGCMG has the same magnitude h_0, the total angular momentum is expressed as

$$\begin{aligned} \boldsymbol{h}_{CMG} &= \boldsymbol{h}_1 + \boldsymbol{h}_2 + \boldsymbol{h}_3 + \boldsymbol{h}_4 \\ &= \boldsymbol{A}[\ h_0 \quad h_0 \quad h_0 \quad h_0\ ]^T, \end{aligned} \tag{5}$$

where $\boldsymbol{A}$ is the transformation matrix from the gimbal frame to spacecraft body frame given by

$$\boldsymbol{A} = \begin{bmatrix} -c\beta \sin\delta_1 & -\cos\delta_2 & c\beta\sin\delta_3 & \cos\delta_4 \\ \cos\delta_1 & -c\beta\sin\delta_2 & -\cos\delta_3 & c\beta\sin\delta_4 \\ s\beta\sin\delta_1 & s\beta\sin\delta_2 & s\beta\sin\delta_3 & s\beta\sin\delta_4 \end{bmatrix}$$

with $c\beta \equiv \cos\beta$, $s\beta \equiv \sin\beta$. The transformation matrix $\boldsymbol{A}$ of SGCMG is in general a function of CMG gimbal angle $\boldsymbol{\delta}$.

Specifically, for SGCMGs, since each flywheel has a constant spinning speed, it is clear that $\boldsymbol{A}\dot{\boldsymbol{h}} = 0$. Moreover, the time derivative of the transformation $\boldsymbol{A}$ is obtained as

$$\dot{\boldsymbol{A}} = \overline{\boldsymbol{A}}\dot{\boldsymbol{\delta}}, \tag{6}$$

where $\overline{\boldsymbol{A}}$ is the Jacobian matrix defined as

$$\overline{\boldsymbol{A}} = \begin{bmatrix} -c\beta\cos\delta_1 & \sin\delta_2 & c\beta\cos\delta_3 & -\sin\delta_4 \\ -\sin\delta_1 & -c\beta\cos\delta_2 & \sin\delta_3 & c\beta\cos\delta_4 \\ s\beta\cos\delta_1 & s\beta\cos\delta_2 & s\beta\cos\delta_3 & s\beta\cos\delta_4 \end{bmatrix}.$$

Therefore, the internal control torque $\boldsymbol{\tau}$ generated by SGCMGs in (3) is reduced to

$$\boldsymbol{\tau} = -h_0\overline{\boldsymbol{A}}\dot{\boldsymbol{\delta}} - \boldsymbol{\omega}^\times \boldsymbol{A}\boldsymbol{h}_{CMG}. \tag{7}$$

Kinematics Equation

The spacecraft kinematics in terms of unit-quaternion can be given by

$$\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{Q} \otimes \boldsymbol{\nu}(\boldsymbol{\omega}) = \frac{1}{2}\begin{bmatrix} \boldsymbol{S}(\boldsymbol{q}) + q_0\boldsymbol{I}_3 \\ -\boldsymbol{q}^T \end{bmatrix}\boldsymbol{\omega}, \tag{8}$$

where $\boldsymbol{Q} = [q_1 \quad q_2 \quad q_3 \quad q_0]^T = [\boldsymbol{q}^T \quad q_0]^T \in \mathbb{Q}_u$ denotes the unit-quaternion describing the attitude orientation of the body frame $\mathcal{B}$ with respect to inertial frame $\mathcal{I}$ and satisfies the constraint $\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1$, $\boldsymbol{\nu}$: $\mathbb{R}^3 \to \mathbb{R}^4$ is defined as the mapping $\boldsymbol{\nu}(\boldsymbol{\omega}) = [\boldsymbol{\omega}^T \ 0]^T$.

Let $\boldsymbol{Q}_d \in \mathbb{Q}_u$ denote the desired attitude. In this paper, the rest-to-rest attitude reorientation problem of rotating a rigid spacecraft from its current attitude $\boldsymbol{Q}$ to a desired attitude $\boldsymbol{Q}_d$ is considered. The unit-quaternion error $\boldsymbol{Q}_e \in \mathbb{Q}_u$ is defined as $\boldsymbol{Q}_e = \boldsymbol{Q}_d^* \otimes \boldsymbol{Q} = [\boldsymbol{q}_e^T \quad q_{e0}]^T$, which describes the discrepancy between the actual unit-quaternion $\boldsymbol{Q}$ and the desired unit-quaternion $\boldsymbol{Q}_d$. The kinematics represented by unit-quaternion error is described as [10]

$$\dot{\boldsymbol{Q}}_e = \frac{1}{2}\boldsymbol{Q}_e \otimes \boldsymbol{\nu}(\boldsymbol{\omega}_e), \tag{9}$$

where $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{R}(\boldsymbol{Q}_e)^T\boldsymbol{\omega}_d$, $\boldsymbol{R}(\boldsymbol{Q}_e)$ is the unit-quaternion error $\boldsymbol{Q}_e$ related rotation matrix [11] defined as $\boldsymbol{R}(\boldsymbol{Q}_e) = (q_{e0}^2 - \boldsymbol{q}_e^T\boldsymbol{q}_e)\boldsymbol{I}_3 + 2\boldsymbol{q}_e\boldsymbol{q}_e^T - 2q_{e0}\boldsymbol{S}(\boldsymbol{q}_e)$, and $\boldsymbol{\omega}_d$ denotes the desired angular velocity. In this paper, since rest-to-rest attitude reorientation problem is only considered, the desired angle velocity is $\boldsymbol{\omega}_d = 0$, which yields $\boldsymbol{\omega}_e = \boldsymbol{\omega}$. Therefore, the attitude error kinematics for rest-to-rest attitude reorientation maneuver in (9) can be rewritten as

$$\dot{\boldsymbol{Q}}_e = \frac{1}{2}\boldsymbol{Q}_e \otimes \boldsymbol{\nu}(\boldsymbol{\omega}) = \frac{1}{2}\begin{bmatrix} \boldsymbol{S}(\boldsymbol{q}_e) + q_{e0}\boldsymbol{I}_3 \\ -\boldsymbol{q}_e^T \end{bmatrix}\boldsymbol{\omega}. \tag{10}$$

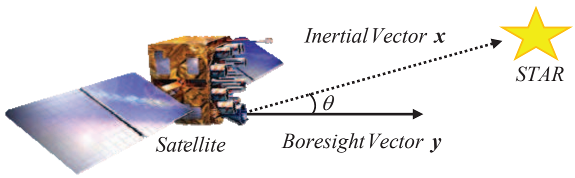

Figure 3. Demonstration of attitude constraint.

Attitude Constraints

Suppose a half-cone angle strictly greater than θ should be maintained between the normalized boresight vector $\boldsymbol{y}$ of the spacecraft instrument and the normalized vector $\boldsymbol{x}$ pointing toward a certain celestial object, as shown in Fig. 3. This means that the cones with an apex angle of θ emanating from the sensitive on-board instruments should exclude the bright objects during the reorientation maneuver. When the attitude of the spacecraft is determined by $\boldsymbol{Q}$, the new boresight vector of the instrument in the inertial coordinates is

$$\boldsymbol{y}_I = (q_0^2 - \boldsymbol{q}^T\boldsymbol{q})\boldsymbol{y} + 2(\boldsymbol{q}^T\boldsymbol{y})\boldsymbol{q} + 2q_0(\boldsymbol{q} \times \boldsymbol{y}). \tag{11}$$

Then the constraints can be expressed by the vector dot product

$$\boldsymbol{x} \cdot \boldsymbol{y}_I < \cos(\theta), \tag{12}$$

Consequently, it follows from (12) that

$$q_0^2\boldsymbol{x}^T\boldsymbol{y} - \boldsymbol{q}^T\boldsymbol{q}\boldsymbol{x}^T\boldsymbol{y} + 2(\boldsymbol{q}^T\boldsymbol{y})\boldsymbol{x}^T\boldsymbol{q} + 2q_0\boldsymbol{q}^T(\boldsymbol{y} \times \boldsymbol{x}) < \cos(\theta) \tag{13}$$

which can be further rewritten as

$$\boldsymbol{Q}^T\begin{bmatrix} \boldsymbol{x}\boldsymbol{y}^T + \boldsymbol{y}\boldsymbol{x}^T - (\boldsymbol{x}^T\boldsymbol{y})\boldsymbol{I}_3 & \boldsymbol{y} \times \boldsymbol{x} \\ (\boldsymbol{y} \times \boldsymbol{x})^T & \boldsymbol{x}^T\boldsymbol{y} \end{bmatrix}\boldsymbol{Q} < \cos(\theta). \tag{14}$$

Suppose there are i constrained objectives associated with the jth on-board sensitive instrument in the spacecraft rotational space. Then, the spacecraft attitude $\boldsymbol{Q} \in \mathbb{Q}_u$ for which the boresight vector $\boldsymbol{y}_j$ with respect to the ith celestial object should satisfy the following constraint

$$\boldsymbol{Q}^T\boldsymbol{M}_i^j\boldsymbol{Q} < \cos(\theta_i^j), \tag{15}$$

where

$$\boldsymbol{M}_i^j = \begin{bmatrix} A_i^j & b_i^j \\ b_i^{jT} & d_i^j \end{bmatrix} \tag{16}$$

with

$$\begin{gathered} A_i^j = \boldsymbol{x}_i\boldsymbol{y}_j^T + \boldsymbol{y}_j\boldsymbol{x}_i^T - (\boldsymbol{x}_i^T\boldsymbol{y}_j)\boldsymbol{I}_3, \\ b_i^j = \boldsymbol{y}_j \times \boldsymbol{x}_i, \quad d_i^j = \boldsymbol{x}_i^T\boldsymbol{y}_j, \\ i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m. \end{gathered} \tag{17}$$

Subsequently, to represent the possible attitude for the jth instrument and the ith celestial object, a subset $\mathbb{Q}_{p_i^j}$ of $\mathbb{Q}_u$ is specified as

$$\mathbb{Q}_{p_i^j} = \{\boldsymbol{Q} \in \mathbb{Q}_u \mid \boldsymbol{Q}^T\boldsymbol{M}_i^j\boldsymbol{Q} - \cos\theta_i^j < 0\}. \tag{18}$$

The angle θ_i^j is the constraint angle about the direction of the ith object specified by $\boldsymbol{x}_i$ for the jth instrument boresight vector $\boldsymbol{y}_j$. Without loss of generality, the domain of the angle θ_i^j for all i and j is restricted to be $(0, \pi)$.

Angular Velocity Constraints

Due to the limited measurement range of the rate gyros or specific mission requirements, constraints on angular velocity might be required. Suppose that angular velocity information is available, then the angular velocity constraints are given by

$$\omega_1 \le \varpi_1,\ \omega_2 \le \varpi_2,\ \omega_3 \le \varpi_3 \tag{19}$$

where ϖ_i $(i = 1, 2, 3)$ is the limitation of allowable operational angular velocities for each axis.

3. Attitude Controller and Steering Logic

Potential Function Design

To avoid unwanted celestial objects and satisfy attitude constraints, the barrier potential $V_a(\boldsymbol{Q})$: $\mathbb{Q}_p \to \mathbb{R}$, is defined as

$$V_a(\boldsymbol{Q}) = \|\boldsymbol{Q}_d - \boldsymbol{Q}\|^2 \sum_{j=1}^{m}\sum_{i=1}^{n} -\alpha \log\left(-\frac{\boldsymbol{Q}^T\boldsymbol{M}_i^j\boldsymbol{Q} - \cos\theta_i^j}{2}\right) \tag{20}$$

where the set $\mathbb{Q}_p = \{\boldsymbol{Q} \in \mathbb{Q}_u \mid \boldsymbol{Q} \in \mathbb{Q}_{p_i^j}\}$ $(i = 1, 2, \ldots, n,$ and $i = 1, 2, \ldots, m)$ represents the possible attitudes of the spacecraft on which the boresight vector of the onboard instruments lie outside of the constrained attitude.

Lemma 1: The potential function in (20) has the following properties [8]:

1. $V_a(\boldsymbol{Q}_d) = 0$
2. $V_a(\boldsymbol{Q}) > 0$, for all $\boldsymbol{Q} \in \mathbb{Q}_p \backslash \{\boldsymbol{Q}_d\}$
3. $\nabla^2 V_a(\boldsymbol{Q}) > 0$ is positive definite for all $\boldsymbol{Q} \in \mathbb{Q}_p$ and $\boldsymbol{Q}_d \in \mathbb{Q}_p$.

The above three properties show that the potential function $V_a(\boldsymbol{Q})$ defined in (20) is smooth and strictly convex for all $\boldsymbol{Q} \in \mathbb{Q}_p$ and $\boldsymbol{Q}_d \in \mathbb{Q}_p$, and it has a global minimum at $\boldsymbol{Q} = \boldsymbol{Q}_d$.

In addition, to satisfy the angular velocity constraints, another potential function is proposed as

$$V_r(\boldsymbol{\omega}) = \frac{1}{2}\log\left[\frac{\prod_{i=1}^{3}\varpi_i^2}{\prod_{i=1}^{3}(\varpi_i^2 - \omega_i^2)}\right]. \tag{21}$$

Attitude Controller Design

The attitude regulation controller is designed as

$$\boldsymbol{\tau} = \boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{J}\boldsymbol{\Upsilon}\left\{-k_1\boldsymbol{\omega} - k_2\boldsymbol{q}_e + \mathrm{Vec}[\nabla V^* \otimes \boldsymbol{Q}]\right\} \tag{22}$$

where the operator Vec[$\cdot$] denotes the vector part of [$\cdot$], and $\boldsymbol{\Upsilon}$ is a diagonal matrix defined as $\boldsymbol{\Upsilon} = \text{diag}\left(\varpi_1^2 - \omega_1^2, \varpi_2^2 - \omega_2^2, \varpi_3^2 - \omega_3^2\right)$.

Consider the following Lyapunov candidate:

$$V_\ell = k_2[\boldsymbol{q}_e^T \boldsymbol{q}_e + (1 - \boldsymbol{q}_{e0})^2)] + V_r(\boldsymbol{\omega}) + 2V_a(\boldsymbol{Q}). \quad (23)$$

The time derivative of V_ℓ is

$$\dot{V}_\ell = k_2 \boldsymbol{\omega}^T \boldsymbol{q} + \boldsymbol{\omega}^T \boldsymbol{\Upsilon}^{-1} \dot{\boldsymbol{\omega}} + \boldsymbol{\nu}(\boldsymbol{\omega})^T (\boldsymbol{Q}^* \otimes \nabla V).$$

Note that

$$\boldsymbol{\nu}(\boldsymbol{\omega})^T (\boldsymbol{Q}^* \otimes \nabla V) = -\boldsymbol{\omega}^T \text{Vec}[(\nabla V^* \otimes \boldsymbol{Q})], \quad (24)$$

it yields

$$\dot{V}_\ell = \boldsymbol{\omega}^T \left(\boldsymbol{\Upsilon}^{-1} \boldsymbol{J}^{-1} (-\boldsymbol{\omega}^\times \boldsymbol{J} \boldsymbol{\omega} + \boldsymbol{\tau}) - \text{Vec}[(\nabla V^* \otimes \boldsymbol{Q})] \right). \quad (25)$$

Substituting the control law (22) into (25) leads to

$$\dot{V}_\ell = -k \boldsymbol{\omega}^T \boldsymbol{\omega}. \quad (26)$$

Therefore, it is clear from (26) that $V_a(\boldsymbol{Q})$ and $V_r(\boldsymbol{\omega})$ are bounded. Consequently, one can obtain that $\ddot{V}_\ell$ is bounded. Hence, according to Barbalat's Lemma, one can conclude that $\lim_{t\to\infty} \boldsymbol{\omega} = 0$. In addition, since the potential $V(\boldsymbol{Q})$ is strictly convex, the following equivalence is ensured

$$\{\boldsymbol{Q} \mid \nabla V_a(\boldsymbol{Q}) = \nabla V_a^*(\boldsymbol{Q}) = 0\} \Leftrightarrow \{\boldsymbol{Q} \mid V_a(\boldsymbol{Q}) = 0\}, \quad (27)$$

which consequently implies that $\lim_{t\to\infty} \boldsymbol{Q}(t) = \boldsymbol{Q}_d$.

In summary, we have the following theorem.

Theorem 1: Consider the spacecraft attitude control systems expressed by (2) and (8) in the presence of attitude constrained zones and angular velocity constraints. The commanded control torque generated by controller (22) guarantees that all closed-loop signals are bounded and that $\lim_{t\to\infty} \boldsymbol{\omega}(t) = 0$ and $\lim_{t\to\infty} \boldsymbol{Q}(t) = \boldsymbol{Q}_d$.

Steering Logic for CMGs

The steering logic transfers the control torques from system level to actuator level. Assuming that the commanded control torque $\boldsymbol{u}$ is computed by a proper attitude controller for achieving the desired three-axis attitude maneuver, then the steering law should be designed such CMGs realize the commanded control torque, i.e., $\boldsymbol{\tau} = \boldsymbol{u}$. One of the major issues in using CMGs for spacecraft attitude control is the CMG geometric singularity problem in which no control torque is generated for the commanded control torque along a particular direction.

Based on the equation in (7), the following relation can be obtained

$$\boldsymbol{\tau} = \boldsymbol{u} = -h_0 \overline{\boldsymbol{A}} \dot{\boldsymbol{\delta}} - \boldsymbol{\omega}^\times \boldsymbol{A} \boldsymbol{h}. \quad (28)$$

To generate the commanded control torque $\boldsymbol{u}$, a steering law to map the commanded signal to gimbal rate is essential. A basic solution of $\dot{\boldsymbol{\delta}}$ for above equation is referred to as the peseudoinverse steering logic, which is given by

$$\dot{\boldsymbol{\delta}} = -\frac{1}{h_0} \overline{\boldsymbol{A}}^\dagger (\boldsymbol{u} + \boldsymbol{\omega}^\times \boldsymbol{A} \boldsymbol{h}), \quad (29)$$

where the peseudoinverse is defined as $\overline{\boldsymbol{A}}^\dagger = \overline{\boldsymbol{A}}^T (\overline{\boldsymbol{A}}\,\overline{\boldsymbol{A}}^T)^{-1}$. As mentioned earlier that the Jacobian matrix $\overline{\boldsymbol{A}}$ is a function of gimbal angle $\boldsymbol{\delta}$, the CMG steering logic may encounter singularity if rank$(\overline{\boldsymbol{A}}) < 3$ for certain sets of gimbal angles.

Several approaches for avoiding or escaping CMGs singular states have been proposed in literature. Here, one of the most commonly used method, the general singular robust (GSR) steering law [12], is employed to handle CMG singularity. According to GSR steering law, the gimbal rate is given by

$$\dot{\boldsymbol{\delta}} = -\frac{1}{h_0} \overline{\boldsymbol{A}}^\sharp \left(\boldsymbol{u} + \boldsymbol{\omega}^\times \boldsymbol{A} \boldsymbol{h}\right), \quad (30)$$

where $\overline{\boldsymbol{A}}^\sharp = \overline{\boldsymbol{A}}^T \left[\overline{\boldsymbol{A}}\,\overline{\boldsymbol{A}}^T + \alpha \boldsymbol{E}\right]^{-1}$, $\alpha = \alpha_0 \exp\left(-\mu m^2\right)$, $m = \sqrt{\det\left(\overline{\boldsymbol{A}}\,\overline{\boldsymbol{A}}^T\right)}$ is the singularity measure, μ is a constant. The matrix $\boldsymbol{E}$ is defined as

$$\boldsymbol{E} = \begin{bmatrix} 1 & \varepsilon_3 & \varepsilon_2 \\ \varepsilon_3 & 1 & \varepsilon_1 \\ \varepsilon_2 & \varepsilon_1 & 1 \end{bmatrix}, \quad \varepsilon_i = \varepsilon_0 \sin(\varpi_i t + \phi_i) \quad (31)$$

where ε_i, ϖ_i and ϕ_i are properly selected.

4. Numerical Simulation

In this section, satellite attitude control with CMGs is studied through simulation. The Kent Ridge 1 (KR-1) satellite is considered as simulation model. Table 1 contains the satellite parameters used for the simulations.

Table 1. Simulation parameters

Parameter	value
Mass (kg)	79
Size (mm)	575×572×384
Moment of inertia (kg·m^2)	diag$\{3.34, 5.29, 3.21\}$

Table 2. Simulation parameters

Constrained Zone (CZ)	Constrained Object	Angle
CZ 1	[0.183 -0.983 -0.036]	30 deg
CZ 2	[0 0.707 0.707]	25 deg
CZ 3	[-0.853 0.436 -0.286]	25 deg
CZ 4	[0.122 -0.140 -0.983]	20 deg

Four SGCMGs in a regular pyramid configuration is used in simulation. The specification of SGCMG is shown in Table 3. The initial gimbal angles are selected as $\boldsymbol{\delta}(0) = [0 \quad 0 \quad 0 \quad 0]^T$ deg, which are far way from singular states.

Table 3. SGCMGs parameters

Parameter	value
Skew angle (deg)	54.74
Maximum momentum (Nms)	$h_{\max} = 2$
Maximum gimbal rate (deg/s)	$\dot{\delta}_{\max} = 40$

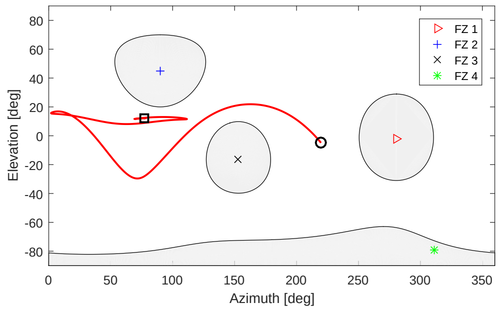

Figure 4. Trajectory of sensitive instrument pointing direction in 2D cylindrical projection.

The parameters in GSR steering law described in equation (30) are selected as:

$$\alpha_0 = 0.01, \quad \mu = 10, \quad \phi_1 = 0, \quad \phi_2 = \frac{\pi}{2}, \quad \phi_3 = \pi,$$
$$\epsilon_i = 0.01 \sin(0.5\pi t + \phi_i), \quad i = 1, 2, 3. \tag{32}$$

The spacecraft is retargeting its sensitive instrument (such as infrared telescopes or interferometers) while avoiding four celestial objects (such as sun light or other bright objects) in the spacecraft reorientation configuration space. Four attitude-constrained zones are chosen without overlapping with each other. The details of the four attitude-constrained zones are given in Table 2, in which the normalized vectors pointing toward the corresponding celestial objects are expressed with respect to the inertial frame. The angular velocity constraints on three axes are required to remain within the range $\varpi_i = 10$ deg/s throughout the attitude control process. Both initial and desired attitude are chosen such that they are out of four attitude-constrained zones. The spacecraft is assumed to have the initial attitude $\boldsymbol{Q}(0) = [0.329\ 0.659\ -0.619\ -0.2726]^T$ and initial angular velocity $\boldsymbol{\omega}(0) = [0\ 0\ 0]^T$ rad/s. The variable α in potential function is chosen as $\alpha = 0.005$. The controller gains in (22) are chosen as $k_1 = 18.2\boldsymbol{J}$ and $k_2 = 20\boldsymbol{J}$.

Simulation Results

The simulation results are shown in Figures 4 to 9. The desired attitude of the flexible spacecraft rotating to is selected as $\boldsymbol{Q}_d = [0.5\ -0.55\ -0.42\ -0.5207]^T$. In Fig. 4, the initial attitude is denoted by $\circ$ and the desired attitude is denoted by $\square$. As shown Fig. 4, the reorientation trajectory generated by the proposed controller in (22) avoids all four constrained zones while achieving the desired attitude. Figs. 5 to 6 describes details of the control performance, where the time histories for attitude error and angular velocity. From Fig. 6, it is clear that the maximum angular velocity is less or equal to the angular velocity limitation. These results show the efficiency of the proposed potential for attitude and angular velocity constraints. Meanwhile, it can also be observed that

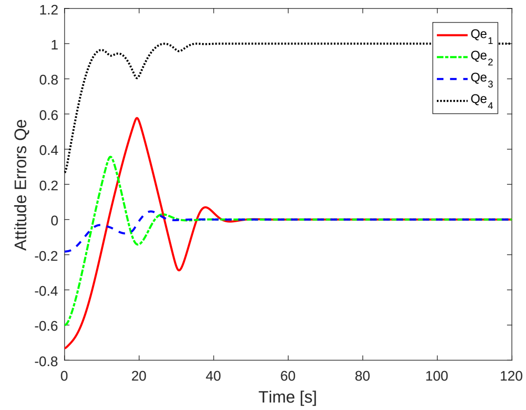

Figure 5. Quaternion error.

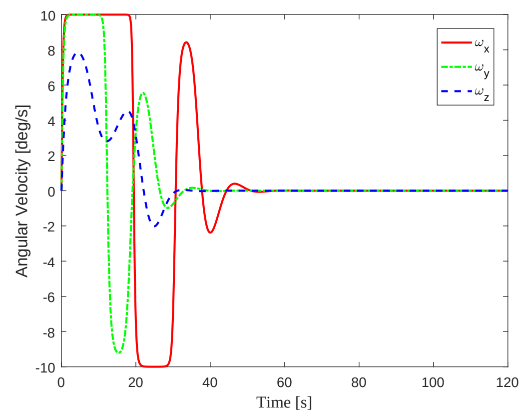

Figure 6. Angular velocity.

the proposed attitude controller in (22) obtains a satisfactory performance in the spacecraft rest-to-rest reorientation despite four attitude-constrained zones and angular velocity constraints. Total angular momentum provided by CMGs are shown in Figure 8. Figure 9 shows the gimbal angle response during the attitude maneuver.

5. SUMMARY

In this paper, a potential function based attitude controller is developed for agile spacecraft to achieve rest-to-rest attitude slew while avoiding the attitude constrained zones . In order to have a fast attitude maneuver, CMG that has a property of large torque amplification is utilized as actuators in attitude control systems. In view of CMG working principle, mathematic models for spacecraft using CMGs are established. A potential function parameterized by unit-quaternion is then proposed with a global minimum at the desired attitude and high potential close to attitude forbidden zones. Nonlinear feedback control law and GSR steering law are formulated to guarantee three-axis attitude control and determine the gimbal rate, respectively. Through numerical simulation on the Kent Ridge 1 satellite, it has shown that fast and high precision attitude pointing maneuver can be achieved using the proposed attitude control systems.

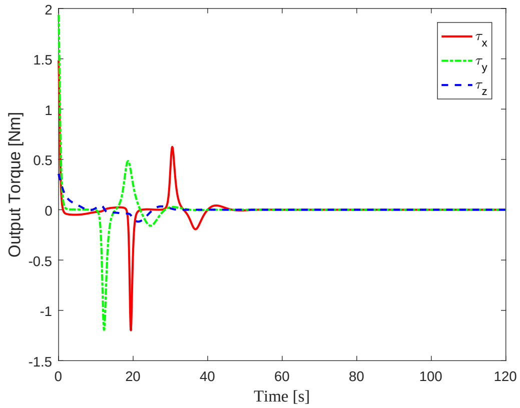

Figure 7. Total control torque generated by CMGs.

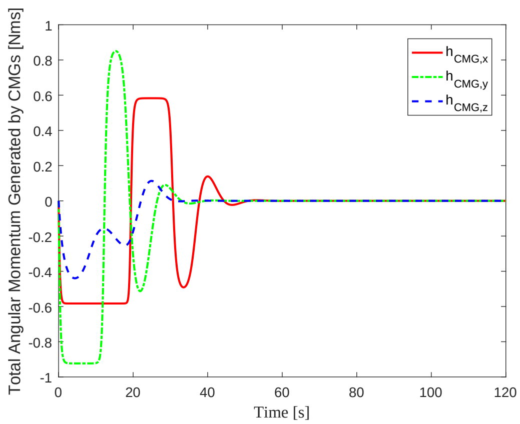

Figure 8. Total angular momentum generated by CMGs.

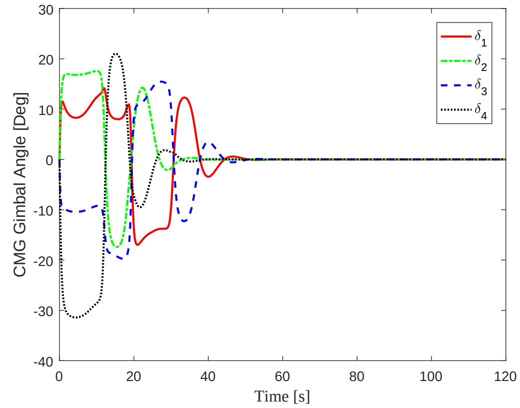

Figure 9. Gimbal angle.

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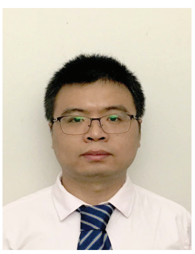

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