Adaptive Robust Fault-Tolerant Attitude Control of Spacecraft with Finite-Time Convergence

Qiang Shen, Danwei Wang, Senqiang Zhu, Eng Kee Poh, and Tianqi Liu
EXQUISITUS, Centre for E-City, School of Electrical and Electronic Engineering
Nanyang Technological University, 639798, Singapore
Email: {qshen1, edwwang, sqzhu, eekpoh, tliu7}@ntu.edu.sg

***Abstract*—This paper aims at investigating finite-time fault-tolerant attitude stabilization control designs for rigid spacecrafts involving two types of actuator faults and modeling uncertainties. In order to express the attitude dynamics in a more convenient manner, the Lagrange-like equation is adopted to describe spacecraft attitude dynamics. Using the terminal sliding mode technique, an on-line adaptive law is employed to estimate the bounds of the uncertainties, and finite-time convergence is achieved by an adaptive fault-tolerant controller in spite of actuator faults. Besides showing fault-tolerant capability, finite-time stability is also guaranteed not only in the reaching phase but also in the sliding phase. Simulation results illustrate the effectiveness of the proposed method.**

I. Introduction

In safety-critical systems, a minor fault in a single component can result in severe performance deterioration or may even produce catastrophic effects. A way to enhance the system reliability and safety in the presence of undesirable faults is by means of fault-tolerant control (FTC) strategies [1], [2], [3]. Generally speaking, the available schemes can be classified into two categories: passive FTC and active FTC. The active FTC approach reacts to the system component malfunctions by reconfiguring the controller based on real-time information from a fault detection and diagnosis (FDD) scheme. Contrary to active approach, passive method utilizes a unique robust controller to deal with all expected faults. In practice, the fault can progress rapidly and the critical system may even become unstable before a desired active fault-tolerant controller can be synthesized [4], [5]. Motivated by the above, this paper concentrates on developing a passive fault-tolerant controller for a spacecraft attitude control system to handle actuator faults.

Several results concerning spacecraft attitude control with actuator faults have been reported [6], [7], [8], [9]. An application of variable structure control techniques to spacecraft attitude stabilization was proposed in [6]. Both passive and active reliable control laws were designed to tolerate the outage of actuators. In [7], by using dynamic inversion and time-delay control theory, a FTC method for attitude tracking control with four reaction wheels was developed. Cai et al. [8] developed an indirect robust adaptive FTC strategy for spacecraft attitude tracking to accommodate modeling uncertainties and thrust faults. In [9], two fault-tolerant adaptive sliding mode control schemes were designed to guarantee global asymptotic convergence of the position tracking error for multiple spacecraft formation flying.

It is well known that a system with finite-time convergence property may ensure higher accuracy, better disturbance rejection properties as well as robustness to uncertainties [10], [11]. Due to these advantages, some effective approaches have been developed on synthesizing finite-time controllers to the spacecraft attitude stabilization problem. In [12], two sliding mode controllers are proposed to achieve finite-time attitude convergence in the presence of model uncertainties and external disturbances. As an extension to [12], fast terminal sliding mode (FTSM) attitude control schemes were presented in [13]. In [14], a finite-time attitude tracking control scheme was derived by using modified FTSM and Chebyshev neural network. Nevertheless, it should be pointed out that these approaches do not explicitly consider actuator faults, and thus may suffer performance degradation in the presence of faults.

In this paper, we shall deal with the problem of compensating for possible actuator failures or faults to stabilize the spacecraft attitude with finite-time convergence. Based on FTSM manifold, the spacecraft attitude dynamics are transformed into a lagrange-like expression with respect to FTSM. With the aid of adaptation mechanism, the bounds of external disturbances, inertia uncertainties, and actuator faults are estimated so that the priori knowledge on these bounds can be relaxed. Consequently, an adaptive FTC approach is proposed which can guarantee the stability of the overall closed-loop systems and finite-time convergence of the spacecraft attitude as well as its time derivative despite the presence of actuator failures or faults. The effectiveness of the proposed FTC schemes are validated by the simulation.

II. Preliminaries

A. Spacecraft Dynamic Equation and Kinematics Equation

The nonlinear differential equations that govern the kinematics and dynamics of the spacecraft in terms of quaternion can be expressed as [8]

$$J(\cdot)\dot{\omega} = -\omega^{\times} J(\cdot)\omega + D\tau + d(t) \tag{1}$$

$$\dot{q}_v = \frac{1}{2}({q_v}^{\times} + q_0 I_3)\omega \tag{2}$$

$$\dot{q}_0 = -\frac{1}{2}{q_v}^T \omega \tag{3}$$

where $J(\cdot) \in \mathbb{R}^{3\times 3}$ is the inertia matrix of the spacecraft, $\omega \in \mathbb{R}^3$ is the angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, $(q_v, q_0) \in \mathbb{R}^3 \times \mathbb{R}$ denotes the unit quaternion describing the attitude orientation of the body frame $\mathcal{B}$ with respect to inertial frame $\mathcal{I}$ and satisfies the constraint $q_v^T q_v + q_0^2 = 1$, $\tau \in \mathbb{R}^n (n > 3)$ is the control torque produced by n actuators, $D \in \mathbb{R}^{3\times n}$ is the actuator distribution matrix, $d(t) \in \mathbb{R}^3$ is the external disturbance. Note that $I_3 \in \mathbb{R}^{3\times 3}$ is the identity matrix and $a^\times \in \mathbb{R}^{3\times 3}$ represents a skew-symmetric matrix for a vector $a = (a_1, a_2, a_3)^T$ given by

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

Here, it is assumed that there exists uncertainty in the matrix $J(\cdot)$, i.e. $J(\cdot) = J_0 + \Delta J$, where J_0 and ΔJ denote the nominal part and uncertain part of $J(\cdot)$ respectively. Then (1) can be rewritten as

$$J_0\dot{\omega} = -\omega^\times J_0\omega + D\tau + d(t) - \Delta J\dot{\omega} - \omega^\times \Delta J\omega \quad (4)$$

For a fault-free case, the output of an actuator is equal to its input. In this paper, two types of actuator faults are considered, namely loss of actuator effectiveness and additive fault (e.g. bias fault). These faults that may be modeled as follows:

$$\tau = E(t)u_c + \overline{u} \quad (5)$$

where u_c is denoted as the input of the actuator, and $E(t) = diag\left(e_1(t), e_2(t), \ldots, e_n(t)\right)$ with $0 \le e_i(t) \le 1 (i = 1, 2, \ldots, n)$, represents the effectiveness of actuators. Note that the case $e_i(t) = 1$ indicates that the ith actuator works normally, $0 < e_i(t) < 1$ implies that the ith actuator partially loses its effectiveness, and $e_i(t) = 0$ denotes complete failure of the ith actuator. $\overline{u} \in \mathbb{R}^n$ represents the actuator fault entering the spacecraft in an additive way. Hence, the attitude dynamic model with actuator faults can be written as

$$J_0\dot{\omega} = -\omega^\times J_0\omega + DE(t)u_c + D\overline{u} + d(t) - \Delta J\dot{\omega} - \omega^\times \Delta J\omega \quad (6)$$

B. Transformed Spacecraft Attitude Dynamics

To express the attitude dynamics in a more convenient manner, the Lagrange-like equation is adopted to describe spacecraft attitude dynamics. Denoting $T = \frac{1}{2}({q_v}^\times + q_0 I_3) \in \mathbb{R}^{3\times 3}$, (2) can be expressed as

$$\omega = P\dot{q}_v \quad (7)$$

with

$$P = T^{-1} = \left[\frac{1}{2}({q_v}^\times + q_0 I_3)\right]^{-1} \quad (8)$$

After taking the time derivative of (7), the following expression for $\dot{\omega}$ is obtained:

$$\dot{\omega} = \dot{P}\dot{q}_v + P\ddot{q}_v \quad (9)$$

Substituting (7) and (9) into (6) and premultiplying both sides of the resulting expression by P^T lead to

$$J^*\ddot{q}_v = -\Xi\dot{q}_v + P^T DE(t)u_c + P^T D\overline{u} + T_d \quad (10)$$

where $J^* = P^T J_0 P$, $\Xi = P^T J_0 \dot{P} - P^T \left(J_0 P\dot{q}_v\right)^\times P$, and $T_d = P^T(d(t) - \Delta J\dot{\omega} - \omega^\times \Delta J\omega)$. T_d is considered as the lumped disturbances and uncertainties. Regarding the dynamic model given in (10), the following properties are given according to [14], [15].

Property 1: The matrix J^* is symmetric positive definite and the matrix $\dot{J}^* - 2\Xi$ satisfies the following skew-symmetric relationship:

$$x^T\left(\dot{J}^* - 2\Xi\right)x = 0 \qquad \forall x \in \mathbb{R}^3 \quad (11)$$

Property 2: The inertia matrix J^* satisfies the following bounded condition:

$$\underline{J}\|x\|^2 \le x^T J^* x \le \bar{J}\|x\|^2 \qquad \forall x \in \mathbb{R}^3 \quad (12)$$

where $\underline{J}$ and $\bar{J}$ are positive constants.

For the development of control law, the following assumptions are given.

Assumption 1: To ensure that P defined in (8) exists, the following condition should be satisfied:

$$det(T) = \frac{1}{2}q_0 \ne 0 \qquad \forall t \in [0, \infty) \quad (13)$$

In order to guarantee that (13) remains valid, it is required that the initial conditions satisfy this constraint, and that the controller is designed to guarantee that $q_0 \ne 0$ for all $t > 0$.

Assumption 2: The lumped term T_d of the disturbances and uncertainties is assumed to satisfy the following condition:

$$\|T_d\| \le \gamma_0\Omega \quad (14)$$

where $\Omega = 1 + \|\omega\| + \|\omega\|^2$ and γ_0 is a positive constant.

Assumption 3: The additive fault introduced in the fault model (5) satisfies

$$\|\overline{u}\| \le f_0 \quad (15)$$

where f_0 is a positive constant.

Assumption 4 [8]: The matrix DED^T is positive definite, and

$$0 < e_0 \le \lambda_{min}(DED^T) \quad (16)$$

where $\lambda_{min}(\cdot)$ represents the minimum eigenvalue of a matrix and e_0 is a positive constant.

Remark 1: The assumption 4 means that, although the n actuators $(n > 3)$ may suffer from partial loss of actuator effectiveness or even complete failure, the number of totally failed actuators is no more than $n-3$ such that DED^T remains positive definite.

C. FTSM Manifold Design

To develop the control scheme, the FTSM manifold $s \in \mathbb{R}^3$ is defined as

$$s = \dot{q}_v + \alpha q_v + \beta sig\left(q_v\right)^r = 0 \tag{17}$$

where $\alpha = diag(\alpha_1, \alpha_2, \alpha_3)$ and $\beta = diag(\beta_1, \beta_2, \beta_3)$ are positive definite and diagonal matrices, r is a positive constant satisfying $0 < r < 1$, $sign(\cdot)$ is the standard sign function, and $sig(\cdot)$ function is defined as

$$sig(x)^r = \left[\, |x_1|^r sign(x_1) \quad |x_2|^r sign(x_2) \quad |x_2|^r sign(x_3) \,\right]^T$$

where x_j is the jth component of x, $j = 1, 2, 3$.

Differentiating s with respect to time yields

$$\dot{s} = \ddot{q}_v + \alpha \dot{q}_v + \beta r\, diag(|q_v|^{r-1})\dot{q}_v \tag{18}$$

Since (18) contains a negative fractional power $r-1$, the singularity will occur if $q_{v,j} = 0$ and $\dot{q}_{v,j} \neq 0$. To avoid singularity, first order derivative of s is modified as [16]

$$\dot{s} = \ddot{q}_v + \alpha \dot{q}_v + \beta q_{vr} \tag{19}$$

with $q_{vr} \in \mathbb{R}^3$ defined as

$$q_{vr,j} = \begin{cases} r|q_{v,j}|^{r-1}\dot{q}_{v,j}, & if\, |q_{v,j}| \geq \epsilon,\, and\, \dot{q}_{v,j} \neq 0 \\ r|\epsilon|^{r-1}\dot{q}_{v,j}, & if\, |q_{v,j}| < \epsilon,\, and\, \dot{q}_{v,j} \neq 0 \\ 0, & if\, \dot{q}_{v,j} = 0 \end{cases} \tag{20}$$

where $q_{vr,j}$ is the jth component of q_{vr}, ϵ is a small positive constant, and $j = 1,\, 2,\, 3$. Then, considering (10), (17), and (19), it can be shown that

$$J^*\dot{s} = -\Xi s + P^T D E u_c + P^T D \bar{u} + F + T_d \tag{21}$$

where $F = \Xi\alpha q_v + \Xi\beta sig(q_v)^r + J^*\alpha\dot{q}_v + J^*\beta q_{vr}$.

Lemma 1 [17]: Suppose $a_1, a_2, \ldots, a_n$ are positive numbers and $0 < p < 2$; then the following inequality holds:

$$(a_1^2 + a_2^2 + \cdots + a_n^2)^p \leq (a_1^p + a_2^p + \cdots + a_n^p)^2$$

Lemma 2 [17]: Assume $\rho_1 > 0$, $\rho_2 > 0$, and $0 < \sigma < 1$; a continuous positive definite function V satisfies the following inequality:

$$\dot{V} \leq -\rho_1 V - \rho_2 V^\sigma \qquad \forall t > 0 \tag{22}$$

then, it can be found that V which starts from $V(0)$ can reach $V = 0$ in finite time. Moreover, the reaching time T_{reach} is given by

$$T_{reach} \leq \frac{1}{\rho_1(1-\sigma)} \ln \frac{\rho_1 V^{1-\sigma}(0) + \rho_2}{\rho_2} \tag{23}$$

Lemma 3: Consider the terminal sliding mode manifold s defined by (17). If the sliding mode manifold satisfies $s = 0$, then the equilibrium point $q_v = 0$ is globally finite-time stable, i.e., the system state q_v which starts from $q_v(0)$ converges to $q_v = 0$ in finite time.

Proof: See the Appendix. ∎

III. Control Design

In this section, we shall show the development of an adaptive finite-time fault-tolerant controller to solve the attitude stabilization problem under actuator failures or faults. In Practice, the bounds on the inertia uncertainties, external disturbances and actuator faults are not always available due to the complicated structure of disturbance, time-varying inertia property, and unexpected characteristics of fault. Therefore, in order to avoid the requirements of a priori knowledge of these bounds, an adaptive mechanism is introduced to estimate these bounds information.

The adaptive control law is designed as

$$\begin{aligned} u_c = D^T P^{-T} \big[-u_{nom} - \hat{\gamma}_1 \left\| P^T D \right\| sign(s) \\ - \hat{\gamma}_2 \left\| u_{nom} \right\| sign(s) \big] \end{aligned} \tag{24}$$

with

$$u_{nom} = k_1 s + k_2 sig(s)^\rho + \|F\|\, sign(s) + \hat{\gamma}_0 \Omega sign(s) \tag{25}$$

where $k_1 = diag(k_{1,1},\, k_{1,2},\, k_{1,3})$ and $k_2 = diag(k_{2,1},\, k_{2,2},\, k_{2,3})$ are two positive definite constant diagonal matrices. The adaptive laws are chosen as

$$\dot{\hat{\gamma}}_0 = c_0 \Omega \|s\|,\, with\, \tilde{\gamma}_0 = \hat{\gamma}_0 - \gamma_0 \tag{26}$$

$$\dot{\hat{\gamma}}_1 = c_1 \left\| P^T D \right\| \|s\|,\, with\, \tilde{\gamma}_1 = \hat{\gamma}_1 - \gamma_1 \tag{27}$$

$$\dot{\hat{\gamma}}_2 = c_2 \|u_{nom}\| \|s\|,\, with\, \tilde{\gamma}_2 = \hat{\gamma}_2 - \gamma_2 \tag{28}$$

where γ_1 and γ_2 are two positive constants satisfying

$$\gamma_1 \geq \frac{f_0}{e_0}, \qquad \gamma_2 \geq \frac{1}{e_0} - 1, \tag{29}$$

$c_0 > 0$, $c_1 > 0$, and $c_2 > 0$ are the design parameters, $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are the estimated value of γ_0, γ_1, and γ_2, respectively.

Theorem 1: Consider the attitude control systems described by (1-3) in the presence of loss of actuator effectiveness fault and additive fault. If the adaptive controller (24) and update laws in (26-28) are applied under Assumptions 1-4, the sliding mode manifold will converge to the neighborhood of $s = 0$ as

$$\|s\| \leq \Delta_s = min\{\Delta_{s1}, \Delta_{s2}\} \tag{30}$$

in finite time, where $\Delta_{s1} = \sqrt{\frac{\bar{J}\phi}{\underline{J}\underline{k}_1}}$, $\Delta_{s2} = \sqrt{\frac{\bar{J}}{\underline{J}}\left(\frac{\phi}{\underline{k}_2}\right)^{\frac{2}{\rho+1}}}$, $\underline{k}_1 = \min_{i=1,2,3}\{k_{1,i}\} > 0$ and $\underline{k}_2 = \min_{i=1,2,3}\{k_{2,i}\} > 0$, and ϕ is a positive constant to be defined later. Furthermore, the spacecraft attitude $q_{v,j}$ and its time derivative $\dot{q}_{v,j}$ will converge to the regions

$$|q_{v,j}| \leq min\left\{\frac{|\Delta_s|}{\alpha}, \left(\frac{|\Delta_s|}{\beta}\right)^{\frac{1}{r}}\right\}, \quad |\dot{q}_{v,j}| \leq 3\Delta_s \tag{31}$$

in finite time, respectively.

Proof: Consider the following Lyapunov function candidate V_2:

$$V_1 = \frac{1}{2}s^T J^* s + \frac{1}{2c_0}\tilde{\gamma}_0^2 + \frac{e_0}{2c_1}\tilde{\gamma}_1^2 + \frac{e_0}{2c_2}\tilde{\gamma}_2^2 \tag{32}$$

Taking the time derivative of V_1, and using (21), leads to

$$\begin{aligned}\dot{V}_1 = &\frac{1}{2}s^T\dot{J}^*s + s^T\left(-\Xi s + P^T DEu_c + P^T D\bar{u} + F + T_d\right)\\ &+ \frac{1}{c_0}\tilde{\gamma}_0\dot{\hat{\gamma}}_0 + \frac{e_0}{c_1}\tilde{\gamma}_1\dot{\hat{\gamma}}_1 + \frac{e_0}{c_2}\tilde{\gamma}_2\dot{\hat{\gamma}}_2 \end{aligned} \tag{33}$$

By substituting the control law (24), adaptive laws (26-28) and Property 1 into (33), it follows that

$$\begin{aligned}\dot{V}_1 \le &- s^T u_{nom} + (1-e_0)s^T u_{nom} - e_0\hat{\gamma}_1\left\|P^T D\right\|\|s\|\\ &- e_0\hat{\gamma}_2\|u_{nom}\|\|s\| + f_0\left\|P^T D\right\|\|s\| + \|F\|\|s\|\\ &+ \|T_d\|\|s\| + \frac{1}{c_0}\tilde{\gamma}_0\dot{\hat{\gamma}}_0 + \frac{e_0}{c_1}\tilde{\gamma}_1\dot{\hat{\gamma}}_1 + \frac{e_0}{c_2}\tilde{\gamma}_2\dot{\hat{\gamma}}_2\\ \le &- s^T k_1 s - s^T k_2 sig(s)^\rho + (1-e_0-e_0\gamma_2)\|u_{nom}\|\|s\|\\ &+ (f_0 - e_0\gamma_1)\left\|P^T D\right\|\|s\| + (\|T_d\| - \gamma_0\Omega)\|s\|\end{aligned}$$

With the help of inequalities (29) and Assumption 2, it can be found that

$$\dot{V}_1 \le -s^T k_1 s - s^T k_2 sig(s)^\rho \le 0 \tag{34}$$

which shows that system (21) is Lyapunov stable.

Note that (34) implies that s, $\tilde{\gamma}_0$, $\tilde{\gamma}_1$, and $\tilde{\gamma}_2$ are bounded. Meanwhile, considering (17), the boundedness of q_v and $\dot{q}_v$ can be derived. Consequently, From (2), (3) and Assumption 1, it can be obtained that $\dot{q}_0$, ω, Ω and P are bounded. Furthermore, if P is bounded, then Ξ and $\|u_{nom}\|$ are also bounded.

Next, in order to prove that the attitude can be stabilized in finite time, another Lyapunov function candidate should be selected. Consider the following Lyapunov function candidate:

$$V_2 = \frac{1}{2}s^T J^* s \tag{35}$$

By differentiating (35) with respect to time, it follows that

$$\begin{aligned}\dot{V}_2 \le &- s^T k_1 s - s^T k_2 sig(s)^\rho + (1-e_0-e_0\gamma_2)\|u_{nom}\|\|s\|\\ &+ (f_0 - e_0\gamma_1)\left\|P^T D\right\|\|s\| + (\|T_d\| - \gamma_0\Omega)\|s\|\\ &- e_0\tilde{\gamma}_2\|u_{nom}\|\|s\| - e_0\tilde{\gamma}_1\left\|P^T D\right\|\|s\| - \tilde{\gamma}_0\Omega\|s\|\end{aligned}$$

With the help of inequalities given in (29) and Assumption 2, it can be shown that

$$\begin{aligned}\dot{V}_2 \le &- s^T k_1 s - s^T k_2 sig(s)^\rho\\ &+ \left(e_0|\tilde{\gamma}_2|\|u_{nom}\| + e_0|\tilde{\gamma}_1|\left\|P^T D\right\| + |\tilde{\gamma}_0|\Omega\right)\|s\|\end{aligned}$$

Because e_0, $|\tilde{\gamma}_0|$, $|\tilde{\gamma}_1|$, $|\tilde{\gamma}_2|$, $\|P\|$,$\|D\|$, $\|u_{nom}\|$, Ω, and $\|s\|$ are all bounded, it can be concluded that $(e_0|\tilde{\gamma}_2|\|u_{nom}\| + e_0|\tilde{\gamma}_1|\left\|P^T D\right\| + |\tilde{\gamma}_0|\Omega)\|s\|$ is bounded. That is, the following inequality will be satisfied:

$$\left(e_0|\tilde{\gamma}_2|\|u_{nom}\| + e_0|\tilde{\gamma}_1|\left\|P^T D\right\| + |\tilde{\gamma}_0|\Omega\right)\|s\| \le \phi \tag{36}$$

where ϕ is a positive constant which represents the upper bound of $(e_0|\tilde{\gamma}_2|\|u_{nom}\| + e_0|\tilde{\gamma}_1|\left\|P^T D\right\| + |\tilde{\gamma}_0|\Omega)\|s\|$. Hence, by using Property 2 and Lemma 1, $\dot{V}_3$ follows that

$$\begin{aligned}\dot{V}_2 \le &- \frac{2\underline{k}_1}{\bar{J}}\sum_{i=1}^{3}\frac{\bar{J}}{2}s_i^2 - \underline{k}_2\left(\frac{2}{\bar{J}}\right)^{\frac{\rho+1}{2}}\left(\sum_{i=1}^{3}\frac{\bar{J}}{2}s_i^2\right)^{\frac{\rho+1}{2}} + \phi\\ \le &- K_1 V_2 - K_2 V_2^{\frac{\rho+1}{2}} + \phi\end{aligned} \tag{37}$$

where $K_1 = \frac{2\underline{k}_1}{\bar{J}} > 0$ and $K_2 = \underline{k}_2\left(\frac{2}{\bar{J}}\right)^{\frac{\rho+1}{2}} > 0$. Thus, (37) can be further written as the following two forms:

$$\dot{V}_2 \le -\left(K_1 - \frac{\phi}{V_3}\right)V_2 - K_2 V_2^{\frac{\rho+1}{2}} \tag{38a}$$

$$\dot{V}_2 \le -K_1 V_2 - \left(K_2 - \frac{\phi}{V_2^{\frac{\rho+1}{2}}}\right)V_2^{\frac{\rho+1}{2}} \tag{38b}$$

From (38a), if $K_1 - \frac{\phi}{V_2} > 0$, a similar structure to (22) in Lemma 2 is kept; therefore the finite-time stability is guaranteed by using Lemma 2. Hence, the region $\|s\| \le \Delta_{s1}$ can be reached in finite time, where Δ_{s1} is defined as

$$\Delta_{s1} = \sqrt{\frac{\bar{J}\phi}{\underline{J}\,\underline{k}_1}} \tag{39a}$$

By using similar analysis for (38b) with Lemma 2, the region $\|s\| \le \Delta_{s2}$ can be reached in finite time, where Δ_{s2} is defined as

$$\Delta_{s2} = \sqrt{\frac{\bar{J}}{\underline{J}}\left(\frac{\phi}{\underline{k}_2}\right)^{\frac{2}{\rho+1}}} \tag{39b}$$

Synthesizing inequalities (39a) and (39b), the sliding mode manifold s will reach the region

$$\|s\| \le \Delta_s = \min\{\Delta_{s1}, \Delta_{s2}\} \tag{40}$$

in finite time.

Moreover, for any s_j in the region Δ_s, we have $|s_j| \le \Delta_s$, $j = 1, 2, 3$. Then, the sliding mode manifold defined in (17) can be written as follows

$$\dot{q}_{v,j} + \alpha q_{v,j} + \beta sig(q_{v,j})^r = \eta_j, \qquad |\eta_j| \le \Delta_s \tag{41}$$

Then, (41) can be written as the following two forms:

$$\dot{q}_{v,j} + \left(\alpha - \frac{\eta_j}{q_{v,j}}\right)q_{v,j} + \beta sig(q_{v,j})^r = 0 \tag{42a}$$

$$\dot{q}_{v,j} + \alpha q_{v,j} + \left(\beta - \frac{\eta_j}{sig(q_{v,j})^r}\right)sig(q_{v,j})^r = 0 \tag{42b}$$

From (42a), if $\alpha - \frac{\eta_j}{q_{v,j}} > 0$, a similar structure to the proposed sliding mode manifold is kept; therefore the finite-time attitude stabilization is guaranteed by using Lemma 3. Furthermore, the attitude $q_{v,j}$ will converge to the region

$$|q_{v,j}| \le \frac{|\eta_j|}{\alpha} \le \frac{\Delta_s}{\alpha} \tag{43a}$$

in finite time. By similar analysis for (42b) and Lemma 3, the attitude $q_{v,j}$ will also converge to the region

$$|q_{v,j}| \le \left(\frac{|\eta_j|}{\beta}\right)^{\frac{1}{r}} \le \left(\frac{\Delta_s}{\beta}\right)^{\frac{1}{r}} \tag{43b}$$

in finite time. Finally, the attitude $q_{v,j}$ will converge to the region

$$|q_{v,j}| \le min\left\{\frac{\Delta_s}{\alpha}, \left(\frac{\Delta_s}{\beta}\right)^{\frac{1}{r}}\right\}$$

in finite time. Moreover, from (41), $\dot{q}_{v,j}$ will converge to the region

$$|\dot{q}_{v,j}| \leq |\eta_j| + \alpha\, |q_{v,j}| + \beta\, |q_{v,j}|^r \leq 3\Delta_s$$

in finite time. This completes the proof. ■

Remark 2: It should be noted that, the control scheme in (24) is independent of the parameters e_0 and f_0. Thus, there is no need to involve a FDD block to identify the actuator faults and the proposed fault-tolerant controller is able to accommodate actuator faults automatically and adaptively whenever the faults take place.

Remark 3: As can be seen from (30), the accuracy of the sliding manifold is related to the parameters k_1 and k_2. Specifically, the larger the parameters $\underline{k}_1$ and $\underline{k}_2$ are, the smaller Δ_s becomes. Also, it is clear in (31) that the final accuracy of attitude stabilization is related to the parameters α, β, and r. It can be concluded that larger α and β and a smaller r lead to better performance.

IV. Numerical Examples and Simulations

To demonstrate the effectiveness and performance of the proposed controller, simulation results are presented in this section. Consider the spacecraft model given in (1-3) with four reaction wheels in a pyramid configuration with the actuator distribution matrix D given as [7]

$$D = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix}$$

The nominal inertia matrix of the spacecraft is $J_0 = diag\,(140, 120, 130)$ kg·m^2. Due to the fuel consumption and onboard payload deployment, the uncertainty in the inertia matrix ΔJ is depicted in Fig. 1 [8]. The initial attitude orientation is chosen as $q_v(0) = [0.3, -0.3, 0.2]^T$ and $q_0(0) = \sqrt{1 - q_v^T q_v}$. The initial angular velocity of the spacecraft is $\omega(0) = [0, 0, 0]^T$ rad/s. The external disturbance model is in the form of $d(t) = (\|\omega\|^2 + 0.005)[sin(0.8t)\ cos(0.5t)\ cos(0.3t)]^T$ Nm. The parameters in FTSM manifold defined in (17) are chosen as $\alpha = 0.05I_3$, $\beta = 0.01I_3$, and $r = 0.6$.

In the context of simulation, the following actuator fault scenario is considered. That is, the first reaction wheel experiences a bias fault with a positive bias torque of 0.2 Nm from 50 s and the second reaction wheel only supplies 50% of its normal actuation power after 5 s. The third reaction wheel loses 60% of its effectiveness in the time interval from 5 s to 40 s, and experiences a bias fault with a negative bias torque 0.2 Nm from 50 s. The fourth reaction wheel fails completely after 10 s.

To illustrate the effectiveness of the proposed adaptive finite-time fault-tolerant controller defined in (24), the attitude stabilization problem under actuator faults in the absence of a priori knowledge of system parameters is simulated. The corresponding parameters of the adaptation law defined in (26-28) are chosen with $c_0 = 0.1$, $c_1 = 4$, and $c_2 = 1$. The initial values of $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are selected as $\hat{\gamma}_0(0) = 0$, $\hat{\gamma}_1(0) = 0$, and $\hat{\gamma}_2(0) = 3$, respectively. In order to eliminate control chattering, the sign function $sign(\cdot)$ in the control law (24) is approximated by a saturation function to smooth the chattering.

The attitude quaternion and time derivative of quaternion are shown in Figs. 2 and 3. It can be seen that the attitude and its derivative converge to the neighborhood of zero in finite time in the presence of inertia uncertainties, time-varying disturbances, and reaction wheel faults. The reaction wheel control torque applied on the spacecraft is shown in Fig. 4. From Fig. 5, we can see that the adaptive parameters $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are bounded, and thus the efficacy of the proposed adaptation laws in (26)-(28) is verified. These simulation results show that fairly good control performance is achieved even under such severe reaction wheel faults.

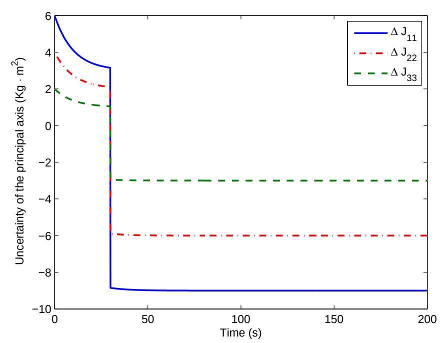

Fig. 1. Uncertainty moment of inertia during spacecraft attitude maneuver.

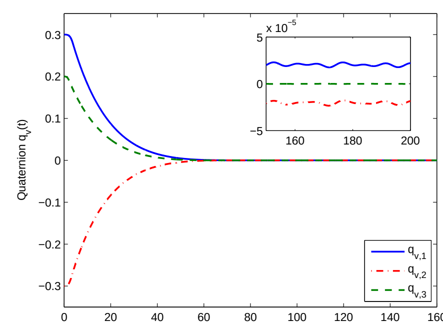

Fig. 2. Quaternion during spacecraft attitude maneuver.

V. Conclusion

In this paper, adaptive finite-time FTC scheme has been investigated for uncertain rigid spacecraft with external disturbances subject to two types of actuator faults. On the base of the terminal sliding mode theory, an adaptive finite-time fault-tolerant controller is developed. The proposed adaptive controller is effective against actuator faults and also guarantees finite-time convergence without requiring a priori knowledge of fault and uncertainty information. The Lyapunov direct approach is employed to prove the stability of the closed-loop system and to show the finite-time convergence of the

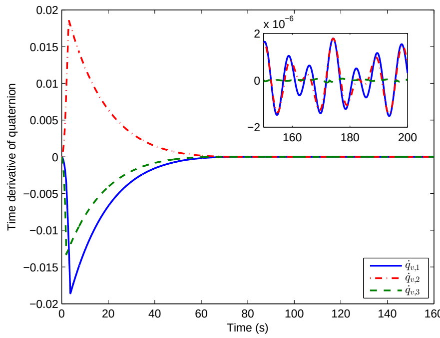

Fig. 3. Time derivative of quaternion during spacecraft attitude maneuver.

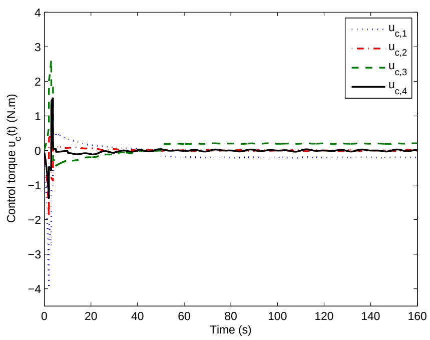

Fig. 4. Four actuation torques (N.m) during spacecraft attitude maneuver.

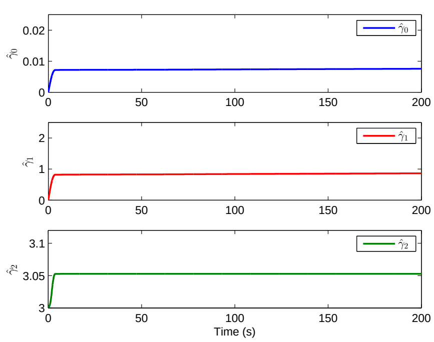

Fig. 5. Adaptive parameters $\hat{\gamma}_0(t)$, $\hat{\gamma}_1(t)$, $\hat{\gamma}_2(t)$ during spacecraft attitude maneuver.

system states. Finally, simulation studies have been presented to illustrate the effectiveness of the proposed finite-time FTC schemes.

Appendix

Proof Lemma 3: In order to prove that q_v can converge to $q_v = 0$ in finite time after the sliding mode manifold $s = 0$ is achieved, a candidate Lyapunov function is constructed as follows

$$V_0 = \frac{1}{2} q_v^T q_v \tag{44}$$

Taking the time derivative of the Lyapunov function (44) alone sliding mode manifold $s = 0$ yields

$$\begin{aligned} \dot{V}_0 &= q_v^T \left(-\alpha q_v - \beta sig(q_v)^r\right) \\ &= -\alpha q_v^T q_v - \beta \sum_{i=1}^{3} |q_{v,i}|^{r+1} \\ &\leq -2\alpha V_0 - 2^{\frac{r+1}{2}} \beta V_0^{\frac{r+1}{2}} \leq -\varrho_1 V_0 - \varrho_2 V_0^{\varsigma} \end{aligned} \tag{45}$$

where $\varrho_1 = 2\alpha$, $\varrho_2 = 2^{\frac{r+1}{2}}\beta$, $\varsigma = \frac{r+1}{2}$, and Lemma 1 is applied. It is clear that (45) has a similar structure to (22), and therefore, using Lemma 2, it can be concluded that q_v converges to origin in finite time.

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