A Design Approach of Model-Based Optimal Fault Diagnosis

Qiang Shen, Lin-Feng Gou, and Xue-Min Yang

School of Power and Energy, Northwestern Polytechnical University,
Xi'an 710072, China
shenqwin@gmail.com

Abstract. The main purpose of fault diagnosis is to detect faults rapidly and accurately, decide the types, sizes and trends of faults, furthermore, separate the fault and make a proper decision to avoid fault. A model-based optimal fault diagnosis method is proposed in this paper. Modeling for a class of typical faults whose dynamic characteristics were known and the initial conditions were unknown with methods in linear system theory, and a reduced-order state observer was designed for fault system. In order to realize fault state of on-line optimal estimation, an optimal fault diagnosis method was proposed by optimal control theory and duality principle, meanwhile, observation error and control energy was optimum. In addition, threshold value is used to decide whether the fault occurred. Simulation results demonstrate the optimal fault diagnosis system can detect the typical faults on-line.

Keywords: fault diagnosis, optimal observer, quadratic performance index, threshold.

1 Introduction

Various environmental changes, unknown disturbances, and changing operating condition are inevitable in many practical dynamical systems, thus sensors, actuators or components failure and faults are very common [1]. A fault [2],[3] in a dynamical system is a deviation of the system structure or the system parameters from the nominal situation. Fault diagnosis should detect system fault accurately, find in which component a fault has occurred, then estimate its magnitude and trend. At present, there are three main methods for fault diagnosis: model-based approach [4], signal processing approach [5],[6] and knowledge-based approach [7],[8].

In Ref. [9], the proportional-integral observer for unknown input descriptor systems is applied to fault estimation; Ref. [10], observer-based fault detection and estimation(FDI) problem using structured residual sets that allow fault isolation; The existence conditions and design algorithm of sliding mode observer for linear descriptor systems with faults are given in Ref. [11]. Ref. [12],[13] proposed nonlinear unknown input observer(UIO)-based FDI approaches, which extended UIO-based FDI from linear system to a respective class of nonlinear system. In this paper, the optimal fault diagnosis problems are studied by using the model-based approach, to a class of faults whose dynamic characteristics are known and the initial conditions are unknown, who realizes the on-line fault diagnosis and makes the observation error and control energy of the designed diagnostic system optimal.

The paper is organized as follows: in Section 2 the problem is formulated, in Section 3 a reduced dimensional observer for linear system is designed, in Section 4 the approach of model-based optimal fault diagnosis is proposed and a threshold value is used to decide whether the fault occurred or not, in Section 5 an example supporting effectiveness of the proposed approach is reported. Finally, some conclusions are given.

2 Problem Formulation

Considering the following linear time-invariant system [14],[15] :

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + D_a f(t) + Gw(t) \\ y(t) = Cx(t) + D_s f(t) + v(t) \end{cases} \tag{1}$$

Where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the measurable input vector, $y(t) \in R^p$ is the output vector, $w(t)$ and $v(t)$ represent the system noise and measurement noise separately. Both of the noises are white Gaussian noise whose statistical property can be described as follows:

$$\begin{cases} E\{w(t)\} = 0 \qquad E\{v(t)\} = 0 \\ E\{w(t)w^T(t+\tau)\} = Q_0\delta(t-\tau) \\ E\{v(t)v^T(t+\tau)\} = R_0\delta(t-\tau) \\ E\{w(t)v^T(t+\tau)\} = 0 \end{cases} \qquad (\forall t, \tau) \tag{2}$$

$f(t) \in R^q$ is the fault vector, which is made up by actuator fault $f_a(t) \in R^{q_a}$ which happens at the time of t_a and sensor fault $f_s(t) \in R^{q_s}$ which happens at the time of t_s, where $q = q_a + q_s$.

$$f(t) = \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix} \tag{3}$$

If $\zeta_a \in R^{r_a}$ is the actuator fault state vector and $\zeta_s \in R^{r_s}$ is the sensor fault state vector, the dynamic behavior of fault is known as

$$\begin{cases} \zeta(t) = 0, \quad t \in [0, t_0) \\ \dot{\zeta}(t) = M\zeta(t), \quad t > t_0, \;\; t_0 = \min\{t_a, t_s\} \\ f(t) = F\zeta(t) \end{cases} \tag{4}$$

Where $\zeta(t) = \begin{bmatrix} \zeta_a(t) \\ \zeta_s(t) \end{bmatrix}, M = \begin{bmatrix} M_a & 0 \\ 0 & M_s \end{bmatrix}, F = \begin{bmatrix} F_a & 0 \\ 0 & F_s \end{bmatrix}$.

Considering the fault system described above, how to design an optimal observer to diagnosis fault is the problem need to be solved.

3 Design of Reduced Dimensional Observer

According to the model of fault system and letting

$$z(t)=\begin{bmatrix} x(t) \\ \zeta(t) \end{bmatrix} \tag{5}$$

Combining (1) and (4), we can get the following state space expression in an augmented form

$$\begin{cases} \dot{z}(t)=A_1 z(t)+B_1 u(t)+G_1 w(t) \\ y(t)=C_1 z(t)+v(t) \end{cases} \tag{6}$$

Where

$$A_1=\begin{bmatrix} A & D_a F \\ 0 & M \end{bmatrix}, B_1=\begin{bmatrix} B \\ 0 \end{bmatrix}, C_1=\begin{bmatrix} C & D_s F \end{bmatrix}, G_1=\begin{bmatrix} G \\ 0 \end{bmatrix}$$

Lemma 1 [15]: The sufficient and necessary condition of completely observable for (C_1, A_1) is (C, A), $\left(\left(C(\lambda I-A)^{-1}D_a F+D_s F\right), M\right)$, (DF, M) are completely observable.

If the system is completely observable, then we can design a reduced dimensional Luenberger observer to detect fault.

Forming a nonsingular matrix $T=\begin{bmatrix} T_1 \\ C_1 \end{bmatrix}=\begin{bmatrix} T_1^T & C_1^T \end{bmatrix}^T$, letting $T^{-1}=\begin{bmatrix} P_1 & P_2 \end{bmatrix}$, then we can get

$$\begin{cases} \overline{w}_c(t)=Tz(t) \\ w_c(t)=T_1 z(t) \end{cases} \Rightarrow \ \overline{w}_c(t)=Tz(t)=\begin{bmatrix} T_1 \\ C_1 \end{bmatrix} z(t)=\begin{bmatrix} w_c(t) \\ y(t)-v(t) \end{bmatrix}$$

$$z(t)=T^{-1}\begin{bmatrix} w_c(t) \\ y(t)-v(t) \end{bmatrix}=P_1 w_c(t)+P_2\left[y(t)-v(t)\right] \tag{7}$$

$$\Rightarrow \begin{cases} \dot{w}_c(t)=T_1 A_1 P_1 w_c(t)+T_1 A_1 P_2 y(t)+T_1 B_1 u(t)+T_1\left(G_1 w(t)-A_1 P_2 v(t)\right) \\ \dot{y}(t)=C_1 A_1 P_1 w_c(t)+C_1 A_1 P_2 y(t)+C_1 B_1 u(t)+C_1\left(G_1 w(t)-A_1 P_2 v(t)\right)+\dot{v}(t) \end{cases} \tag{8}$$

Introducing the equivalent input $\overline{u}(t)$ and output $\overline{y}(t)$

$$\begin{aligned} &\overline{u}(t)=T_1 A_1 P_2 y(t)+T_1 B_1 u(t)+T_1\left(G_1 w(t)-A_1 P_2 v(t)\right) \\ &\overline{y}(t)=\dot{y}(t)-C_1 A_1 P_2 y(t)-C_1 B_1 u(t)-C_1\left(G_1 w(t)-A_1 P_2 v(t)\right)-\dot{v}(t)=C_1 A_1 P_1 w_c(t) \end{aligned} \tag{9}$$

Then we can get

$$\begin{cases} \dot{w}_c(t) = T_1 A_1 P_1 w_c(t) + \overline{u}(t) \\ \overline{y}(t) = C_1 A_1 P_1 w_c(t) \end{cases} \tag{10}$$

If $\left(C_1 A_1 P_1, T_1 A_1 H_1\right)$ is completely observable, reduced dimensional Luenberger observer can be formed as follow:

$$\dot{\hat{w}}_c(t) = \left(T_1 - LC_1\right) A_1 P_1 \hat{w}_c(t) + \overline{u}(t) + L\overline{y}(t) \tag{11}$$

Where L is observer gain, combining (9) and (11), we can get

$$\dot{\hat{w}}_c(t) = (T_1 - LC_1)\left[A_1 P_1 \hat{w}_c(t) + B_1 u(t) + A_1 P_2 y(t) + \left(G_1 w(t) - A_1 P_2 v(t)\right)\right] + L\left[\dot{y}(t) - \dot{v}(t)\right] \tag{12}$$

Using a special variable transformation

$$x_c(t) = \hat{w}(t) - L\left[y(t) - v(t)\right]$$

Then

$$\begin{aligned} \dot{x}_c(t) &= \dot{\hat{w}}(t) - L\left[\dot{y}(t) - \dot{v}(t)\right] \\ &= (T_1 - LC_1)\left[A_1 P_1 \hat{w}_c(t) + B_1 u(t) + A_1 P_2 y(t) + \left(G_1 w(t) - A_1 P_2 v(t)\right)\right] \end{aligned} \tag{13}$$

From (7) we can get

$$\hat{z}(t) = P_1 \hat{w}_c(t) + P_2\left[y(t) - v(t)\right] = P_1 x_c(t) + (P_1 L + P_2)\left[y(t) - v(t)\right] \tag{14}$$

Letting

$$P_1 = \begin{bmatrix} P_{11} \\ P_{21} \end{bmatrix}, P_2 = \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix}$$

Finally, the reduced dimensional Luenberger observer is

$$\begin{cases} \dot{x}_c(t) = (T_1 - LC_1)\left[A_1 P_1 x_c(t) + B_1 u(t) + G_1 w(t) + A_1\left(P_1 L + P_2\right)\left(y(t) - v(t)\right)\right] \\ \hat{f}(t) = F\left[P_{21} x_c(t) + (P_{22} + P_{21} L)\left(y(t) - v(t)\right)\right] \end{cases} \tag{15}$$

4 Design of Optimal Fault Diagnosis

In order to realize optimal fault diagnosis, using the reduced dimensional observer designed above, observer gain L must satisfy a quadratic performance index to fulfill the observation error and control energy of the designed diagnostic system optimal. A new design approach is proposed by optimal control theory and duality principle.

Define observation error as

$$\tilde{w}_c = w_c - \hat{w}_c \tag{16}$$

Using (**8**) compare to (**12**), observation error equation is written to be

$$\dot{\tilde{w}}_c = \dot{w}_c - \dot{\hat{w}}_c = T_1 A_1 P_1 \tilde{w}_c - LC_1 A_1 P_1 \tilde{w}_c \tag{17}$$

So, we can get the dual system of observation error equation

$$\dot{\psi}(t) = \left(T_1A_1P_1\right)^T \psi(t) - \left(C_1A_1P_1\right)^T L^T\psi(t) \tag{18}$$

Obviously, the dual system (**17**) and error equation (18) have the same eigenvalues

$$\det\left[\lambda I - (T_1A_1P_1)^T + (C_1A_1P_1)^T L^T\right] = \det\left[\lambda I - T_1A_1P_1 + LC_1A_1P_1\right] \tag{19}$$

Above all, we can design an optimal state feedback gain matrix L of the dual system to achieve the observer optimal.

Letting

$$\varphi(t) = -L^T\psi(t) \tag{20}$$

Where, L^T is defined as the equivalent state feedback of dual system. So, the dual system can be written as the following open loop system

$$\dot{\psi}(t) = \left(T_1A_1P_1\right)^T \psi(t) + \left(C_1A_1P_1\right)^T \varphi(t) \tag{21}$$

In (21), $\varphi(t)$ is known as the equivalent input vector.

Taken together, the optimal estimate of fault diagnosis system (15) has transformed into optimal states feedback of the dual system (21).

For the dual system, considering the quadratic performance index as follow

$$J = \lim_{t\to\infty}\frac{1}{T}\int_0^T \left(\psi^T(t)Q_{obs}\psi(t) + \varphi^T(t)R_{obs}\varphi(t)\right)dt \tag{22}$$

Where, Q_{obs} is positive semidefinite matrix and R_{obs} is positive definite matrix. In (22), $\psi^T(t)Q_{obs}\psi(t)$ is observation error and $\varphi^T(t)R_{obs}\varphi(t)$ is the control energy. In a word, the quadratic performance index (22) makes the system in the whole process of dynamic observation error and consumption of control energy satisfies a tradeoff optimal.

Based on optimal control theory, we can get optimal control rule of the dual system,

$$\varphi^*(t) = -R_{obs}^{-1}(C_1A_1P_1)P_o(t)\psi(t) \tag{23}$$

Where, $P_o(t)$ is the solution of the following Riccati equation,

$$\dot{P}_o(t) = -P_o(t)\left(T_1A_1P_1\right)^T - T_1A_1P_1P_o(t) + P_o(t)\left(C_1A_1P_1\right)^T R_{obs}^{-1}C_1A_1P_1P_o - Q_{obs} \tag{24}$$

Consequently, the state feedback matrix of dual system in the sense of optimum is known as

$$L^T = R_{obs}^{-1}(C_1A_1P_1)P_o(t) \tag{25}$$

Then, the state feedback gain matrix of optimal fault diagnosis system is the inversion of (25)

$$L = P_o(t)(C_1A_1P_1)^T R_{obs}^{-1} \tag{26}$$

For the detected fault, threshold value can be confirmed based on the principle of maximum inconsistent [16]. Considering each unit which may have some fault, the basic idea is to determine allowed deviation under the worst situations, and according to the deviation to set threshold value of fault diagnosis system.

As for a system with two redundant channels, each unit which may have some fault with an allowable error E_i $(i=1,2,\cdots,n)$, and the gain of each unit is R_i $(i=1,2,\cdots,n)$, therefore, the maximum deviation can be described as

$$E_{\max} \approx \sum_{i=1}^{n}\left(|E_i|+|E_i|\right) = 2\sum_{i=1}^{n}|E_i| \tag{27}$$

Finally, we can define the threshold value THR as

$$THR \geq E_{\max} \tag{28}$$

For a fault system, if the final diagnosed results beyond the threshold periodically, we can decide the fault has happened.

5 Application Example

Considering the nominal model of aero-engine in the condition of $H=0\text{km}$, $Ma=0$,the input vector is $u(t)=\left[q_{m,f}(t) \quad A_8(t)\right]^T$,state vector is $x(t)=\left[N_H \quad N_L\right]^T$,output vector is $y(t)=\left[N_H \quad \pi_P\right]^T$,where $q_{m,f}(t)$ is fuel flux, $q_{m,f}(t)$ is nozzle area, N_H and N_L are respectively engine high-pressure rotation speed and low-pressure rotation speed, π_P is the pressure ratio.

Establishing expand object model, considering the fuel metering device and tail spout actuator cylinder as inertial element, the characteristic time is 0.05s and 0.1s, their transfer function can be written as G_1 and G_2,

$$G_1=\frac{1}{0.05s+1}=\frac{20}{s+20},\ G_2=\frac{1}{0.1s+1}=\frac{10}{s+10}$$

$\dot{q}_{m,f}$ and $\dot{A}_8$ can also be seen as state vector, then the extended object model is described as following [17]:

$$\dot{x}_{ex}(t)=\begin{bmatrix} -6.715 & 2.256 & 0.361 & 0.442 \\ 7.380 & -9.089 & -0.304 & 2.032 \\ 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & -10 \end{bmatrix} x_{ex}(t)+\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 10 \end{bmatrix} u(t)+\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} f(t)+\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} w(t)$$

$$y(t)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.473 & 2.320 & 0.371 & -0.717 \end{bmatrix} x_{ex}(t)+\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} f(t)+v(t)$$

Where $\dot{x}_{ex}=\left[n_H \quad n_L \quad \dot{q}_{m,f} \quad \dot{A}_8\right]^T$, $y(t)=\left[N_H \quad \pi_P\right]^T$, $u(t)=\left[q_{m,f}(t) \quad A_8(t)\right]^T$, the fault model is

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The sensor fault is a sine wave fault, which happens at the time of $t_s = 20s$, and the frequency is $w_s = 2\,rad/s$; the actuator fault is also a sine fault, which happens at the time of $t_a = 30s$, and the frequency is $w_a = 1\,rad/s$, the amplitude of white Gaussian noise $w(t), v(t)$ are 0.2.

In optimal fault diagnosis system $T_1 = [0 \quad I_6]$, in quadratic performance index (**22**), $R_{obs} = 1$ and $Q_{obs} = I_6$, the allowable error of sensor is $E_s = 10\%$,and the allowable error of actuator is $E_a = 10\%$, based on the principle of maximum inconsistent we can get the maximum deviation is $E_{\max} = 10\% + 10\% = 20\%$. Hence, the threshold is $THR > 20\%$.

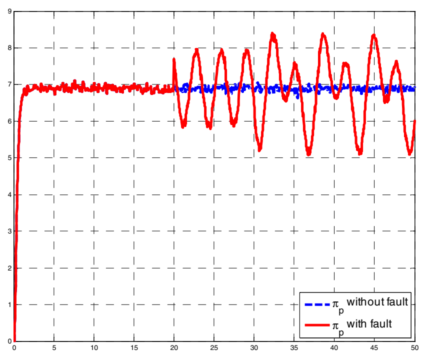

Fig. 1. System output π_P

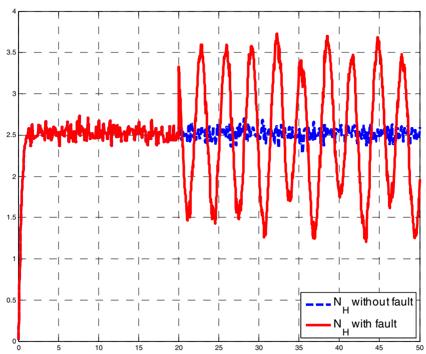

Fig. 2. System output N_H

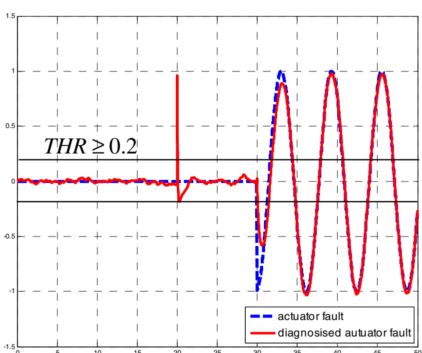

Fig. 3. True value and estimated value of actuator fault

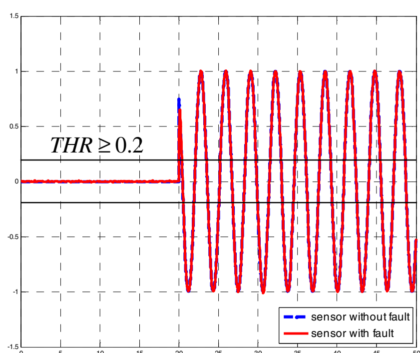

Fig. 4. True value and estimated value of sensor fault

As for the input is step signal $u(t) = [10 \quad 10]^T$, we can get the optimal observer gain is

$$L = \begin{bmatrix} \text{-0.0090} & 0.0525 & 0.0437 & 0.2748 & \text{-0.9590} & 0.8423 \\ \text{-0.0769} & 0.1554 & 1.2829 & 0.5260 & 0.0769 & \text{-0.7671} \end{bmatrix}^T$$

Figure 1 and figure 2 are the system output π_P and N_H in the situation of happening fault and without fault. From figure 1 and figure 2, we can see dynamic performance has changed a lot after fault happens. In simulation, we set the threshold value $THR \geq 20\%$, through comparing the curves of true value and estimated value about sensor fault and actuator fault, we can see the diagnosed curves beyond the threshold periodically, actually, so we can judge the fault has happened. In figure 3, we can get the conclusion the actuator fault happens almost in the 30 seconds, similarly, in figure 4, the sensor fault happens almost in the 20 seconds.

6 Conclusion

A model-based optimal fault diagnosis method of aero-engine is proposed in this paper. Modeling for a class of typical faults whose dynamic characteristics are know and the initial conditions are unknown, with methods in linear system theory, a reduced-order state observer is designed for fault system. In order to realize fault state of on-line optimal estimation, an optimal fault diagnosis is proposed by optimal control theory and duality principle, at the same time, observation error and control energy are optimum. In addition, a threshold value is used to decide whether the fault occurred. Simulation results demonstrate the optimal fault diagnosis system can follow the fault on-line and ensure the following error smallest.

References

1. Liu, N., Zhou, K.: Optimal Robust Fault detection for Linear Discrete Time Systems. Journal of Control Science and Engineering (2008)
2. Chen, J., Patton, R.J.: Robust Model-based Fault diagnosis for Dynamic System. Kluwer Academic Publishers, Dordrecht (2009)
3. Mogens, B., Michel, K., Jan, L., Marcel, S.: Diagnosis and Fault-Tolerant Control. Springer, Heidelberg (2006)
4. He, H., Wang, G.Z., Ding, S.X.: A new parity space approach for fault detection based on stationary wavelet transform. IEEE Transactions on Automatic Control 49(2), 281–287 (2004)
5. Sun, R.X., Tsung, F., Qu, L.S.: Evolving kernel principal component analysis for fault diagnosis. Computers and Industrial Engineering 53(2), 361–371 (2007)
6. He, H., Ding, S.X., Wang, G.Z.: Integrated design of fault detection systems in time-frequency domain. IEEE Transactions on Automatic Control 47(2), 384–390 (2002)
7. Rajakarunakaran, S., Venkumar, P., Devaraj, D., et al.: Artificial neural network approach for fault detection in rotary system. Applied Soft Computering 8(1), 740–748 (2008)

8. Papadopoulos, Y.: Model-based system monitoring and diagnosis of failures using statecharts and fault trees. Reliability Engineering and System Safety 81(3), 325–341 (2003)
9. Damien, K.: Unknown Iinput Proportional Multiple-Integral Observer Design for Linear Descriptor Systems: Application to State and Fault Estimation. IEEE Transaction on Automatic Control 5(2), 212–217 (2005)
10. Commault, C., Dion, J.-M., Sename, O., Motyeian, R.: Observer-Based Fault Detection and Isolation for Structured Systems. IEEE Transactions on Automatic Control 47(12), 2074–2079 (2002)
11. Yu, J.-Y., Liu, Z.-Y.: Fault Reconstruction Based on Sliding Mode Observer for Linear Descriptor Systems. In: Proceedings of the 7th Asian Control Conference, Hong Kong, China, pp. 1132–1137 (August 2009)
12. Seliger, R., Frank, P.M.: Fault Diagnosis by Disturbance Decoupled Nonlinear Observers. In: Proc. IEEE CDC, Brighton, UK, pp. 2248–2253 (1991)
13. Seliger, R., Frank, P.M.: Robust component fault detection and isolation in nonlinear dynamic systems using nonlinear unknown input observers. In: Proc. IFAC/IMACS Symp. SAFEPROCESS 1991, Baden-Baden, Germany, pp. 313–318 (1991)
14. Li, J., Tang, G.-Y., Gao, H.-W.: Fault Detection and Self-Restore Control for Linear Systems. In: The Proceedings of Sixth International Conference on Intelligent Systems Design and Applications, pp. 873–878. IEEE Computer Society, Jinan (2006)
15. Ye, R.-H.: Study on Observer-Based Fault Diagnosis and Optimal Fault-Tolerant Control Approaches, Qingdao Agriculture University, China (2008)
16. Wen, X., Zhang, H.-Y., Zhou, L.: Fault Diagnosis and Fault-Tolerant Control for Control system. China Machine Press, Beijing (1998)
17. Fan, S.-Q., Li, H.-C., Fan, D.: Aero-engine control. Northwestern Polytechnic University Press, Xi'an (2008)