Fault Estimation and Fault-Tolerant Steering Law for Single Gimbal Control Moment Gyro Systems

Chengfei Yue, Qiang Shen, Goh Cher Hiang and Tong Heng Lee
Department of Electrical and Computer Engineering
National University of Singapore, Singapore
chengfei_yue@u.nus.edu; qiang.shen@hotmail.com; elegch@nus.edu.sg;eleleeth@nus.edu.sg

***Abstract*—Single gimbal control moment gyros (SGCMGs) have been widely used on agile satellites to get a rapid retargeting capability. To enhance the reliability and safety of the SGCMG system, this paper addresses the fault estimation and fault-tolerant steering problem. The SGCMG is modeled as a two-loop system including a wheel speed control loop and a gimbal rate control loop, and a cascade multiplicative fault model of SGCMG is developed. Then, in view of the complexity of the gimbal fault, a local adaptive fault estimator is proposed to reconstruct the total time-varying fault effects for each SGCMG. Using the estimated fault effects, a fault-tolerant steering logic is further developed to not only allocate the commanded attitude control torque properly but also compensate the fault effects. To verify the proposed fault estimator and fault-tolerant steering logic, numerical simulations are carried out on a SGCMG-actuated spacecraft.**

***Index Terms*—SGCMG, fault estimation, fault-tolerant control, steering law**

I. Introduction

Single gimbal control moment gyros (SGCMGs) have been studied extensively [1], [2], [3] and have been applied to spacecraft attitude control and momentum management for various space missions, such as Pleiades and Wordview series. In practical space missions, redundant momentum exchange devices are employed to enhance reliability. Even so, faults or failures may occur occasionally. For instance, CMG #1 and CMG #2 installed on the Skylab failed in 2002 [4]. These faults or failures may lead to performance deterioration or even mission failure and some catastrophic consequence. Thus developing a fault-tolerant control strategy to accommodate the SGCMG fault and maintain a satisfactory control performance is of significant importance.

For the spacecraft fault-tolerant control (FTC) problem, various model-based approaches for the reaction wheel (RW) actuated spacecraft have been proposed. In [5], [6], [7] and [8], a mathematical model consisting of an effectiveness gain matrix and a bias vector is proposed to represent four different kinds of RW fault. In contrast, the fault-tolerant control results for SGCMG-actuated spacecraft is rare. In [9], the potential faults are summarized but the control effect caused by the faults are not evaluated mathematically. In [10] and [11], the gimbal rate command is designed directly to accommodate the gimbal fault. However, the fault model is not established. In this paper, to have a comprehensive fault model for SCCMG, the mechanism of the SGCMG is analyzed and a cascade multiplicative fault model of SGCMG is given.

After establishing the fault model, the fault effects should be estimated such that a fault-tolerant strategy can be developed to accommodate the fault. In [12], an indirect fault estimator was developed to reconstruct the fault. Using the reconstructed fault information, a fault-tolerant controller was proposed. In [13], Boskovic and Mehra proposed that failures of flight control actuator are generally localized to the actuator dynamics only. Based on this foundation, decentralized local failure detection and identification (FDI) observers were designed to estimate failure-related parameters for each of the actuators. Then a intergraded FDI-FTC algorithm was developed for a nonlinear system actuated by actuators that possess a higher-order actuator dynamics and may experience several different types of failures. In [14], local adaptive observer and finite-time fault-tolerant controller were also designed to handle RW faults. Inspired by the work of [13] and [14], we design local adaptive estimators to reconstruct total fault effects in the gimbal rate control loop for each SGCMG, rather than to estimate the total resultant error torque caused by the whole actuator cluster as in [6] and [12]. Then the reconstructed information is used to develop fault-tolerant steering law for accommodating the gimbal fault. The proposed local fault estimator is simple to implement, while being flexible with respect to the possibility of multiple kinds of faults occurring simultaneously or sequentially. In addition, we compensate the SGCMG fault in the steering logic design instead of redesigning the attitude controller that are commonly used for accommodating faults in traditional fault-tolerant control systems. That is, our approach handles the SGCMG faults without reconfiguring the attitude controller, which avoids the controller switching transient in existing fault-tolerant attitude control methods, such as [12] and [14].

The remaining part of this paper is organized as follows. Section II presents the mathematical fault model of SGCMG. Section III demonstrates the proposed local fault estimator. The fault-tolerant steering logic is proposed in Section IV. In Section V, numerical simulation on a rigid spacecraft using four SGCMGs is carried out to verify the proposed fault estimation method and fault-tolerant steering logic. Finally, this paper ends with the conclusion in Section VI.

II. SGCMG FAULT MODEL

The SGCMG contains a spinning rotor mounted on a gimbal frame. In nominal condition, the rotor hold a constant speed using a brushless DC (BLDC) motor, while the gimbal is manipulated to change the direction of angular momentum by a stepper motor. The stepper motor provides precise gimbal control of CMGs and the BLDC motor offers an efficient way of driving the momentum wheel to store the angular momentum [9]. Then a gyroscopic reaction torque orthogonal to both the rotor spin and gimbal axes is generated. With a small input of the gimbal, a much larger control torque can be produced to act on the spacecraft. Specifically, the torque is proportional to both the angular momentum and gimbal angular rate and can be calculated as:

$$\boldsymbol{t} = -h_0 \dot{\delta} \hat{\boldsymbol{t}}, \tag{1}$$

where h_0 is the constant angular momentum of the spinning rotor, $\dot{\delta}$ is the gimbal rate and $\hat{\boldsymbol{t}}$ is a unit vector in the direction of output torque.

For a SGCMG, the flywheel control loop is to hold a constant h_0 and the gimbal control loop is to generate the gimbal rate $\dot{\boldsymbol{\delta}}$. Both of the two loops contain an electric motor (EM) and the corresponding variable speed drive (VSD) system. The potential fault of a SGCMG may occur in the mechanical and/or electrical system of the EM and sensors and actuators of the VSD in either rotor control loop or the gimbal rate control loop.

A. Fault model of an EM-VSD system

The details about potential faults in EM-VSD system can be found in [9], [15], [16], [17]. Specially, for the EM, potential faults are categorized into:

- stator faults;
- rotor faults;
- eccentricity-related faults; and
- bearing and gearbox faults or failures.

With regard to the VSD, the faults are classified into:

- sensor faults; and
- actuator (actuator in VSD) faults.

Generally speaking, the component fault of the EM will influence the system matrix of the EM in its state-space representation. Consequently, they can be modeled in a multiplicative way. For the sensors such as Hall position sensor and electrical tachometer and the inverters act as actuator in VSD, they can be modeled in an additive way. Then, based on the result in [18], the SGCMG fault model can be given as follows:

$$\begin{cases} \Omega = \eta^{\Omega}\Omega_c + \Omega_o, & \text{Rotor speed control loop} \\ \dot{\delta} = \eta^{g}\dot{\delta}_c + \dot{\delta}_o, & \text{Gimbal rate control loop} \end{cases} \tag{2}$$

where Ω and Ω_c are rotating speed of the flywheel and its command input, $\dot{\delta}$ and $\dot{\delta}_c$ are gimbal rate output and its command from SGCMG steering law, η^{Ω} and η^{g} are effectiveness gains satisfying $0 \le \eta^{\Omega} \le 1$ and $0 \le \eta^{g} \le 1$, Ω_o and $\dot{\delta}_o$ are bounded offsets of wheel speed and gimbal rate.

B. SGCMG fault model

As mentioned before, the potential faults may locate in the rotor speed control loop and/or gimbal rate control loop. For the rotor control system, the angular momentum is the product of its inertia J_{Ω} and the rotor angular velocity Ω, i.e. $h_0 = J_{\Omega}\Omega$. With consideration of the possible faults in rotor speed control loop, which is modeled in (2), the rotor momentum subject to faults is given by

$$h_0 = J_{\Omega}\left(\eta^{\Omega}\Omega_c + \Omega_o\right). \tag{3}$$

For the gimbal rate control loop of a SGCMG, its fault can be modeled as in (2):

$$\dot{\delta} = \eta^{g}\dot{\delta}_c + \dot{\delta}_o \tag{4}$$

As stated in [19], the dynamics of CMG gimbal is independent of rotor momentum for the case of a very stiff gimbal. Then the fault model of the SGCMG can be obtained by substituting (3) and (4) into (1):

$$\boldsymbol{t} = -J_{\Omega}\left(\eta^{\Omega}\Omega_c + \Omega_o\right)\left(\eta^{g}\dot{\delta}_c + \dot{\delta}_o\right)\hat{\boldsymbol{t}}. \tag{5}$$

For the SGCMGs, faults in the rotor control loop can be easily detected, and its consequence is equivalent to replacing the SGCMG by a smaller one that generates less torque. However, as the gimbal fault has a nonlinear effect to the torque generated by the SGCMGs, it cannot be handled straightforwardly like the rotor fault. We will develop local estimator for each SGCGM to estimate the overall fault effect in next section.

III. FAULT ESTIMATION FOR SGCMG

In practical space missions, multiple SGCMGs are employed to generate the commanded torques from attitude controller. Here, we assume that N SGCMGs ($N \ge 3$) are equipped in the spacecraft, and each of N SGCMGs may encounter faults. To obtain the fault information in each SGCMG, we develop a local fault estimator for each SGCMG. The overall attitude control system with local fault estimation is demonstrated in Fig. 1.

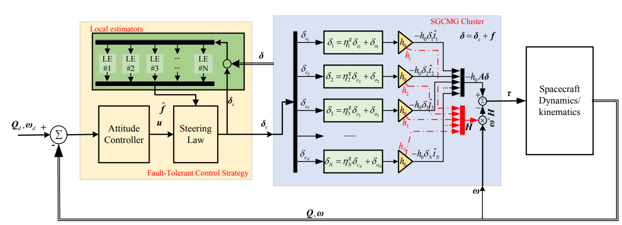


Fig. 1: Overall attitude control system with LEs

A. Local Fault Estimator

In light of (4), the gimbal fault model can be further rewritten as

$$\dot{\delta} = \dot{\delta}_c + f. \tag{6}$$

where $\dot{\delta}_c$ is the gimbal rate command, $f = (\eta^g - 1)\dot{\delta}_c + \dot{\delta}_o$ denotes the total fault effect consisting of the loss of effectiveness fault and additive offset. This total fault effect f in each SGCMG is assumed to be differentiable in the sense that its time derivative is bounded with a positive constant $\overline{f}$, i.e., $|\dot{f}| \le \overline{f}$.

To estimate the total fault effect f in a SGCMG, we define an auxiliary variable as [12], [20]

$$\phi = f - k\delta, \tag{7}$$

where k is a positive constant. Next, the local adaptive estimator for fault estimation is proposed as follows:

$$\dot{\hat{\delta}} = \dot{\delta}_c + \alpha(\delta - \hat{\delta}) + \hat{f}, \tag{8}$$

$$\dot{\hat{\phi}} = -k(\dot{\delta}_c + \hat{\phi} + k\hat{\delta}), \tag{9}$$

where α is a positive design parameter. Define estimation errors $\tilde{\delta} = \delta - \hat{\delta}$ and $\tilde{\phi} = \phi - \hat{\phi}$. Based on the definition of the auxiliary variable in (7), the fault estimation error can be expressed as $\tilde{f} = \tilde{\phi} + k\tilde{\delta}$. Then, the estimation error dynamics of gimbal angle and the auxiliary variable can be obtained as

$$\dot{\tilde{\delta}} = -(\alpha - k)\tilde{\delta} + \tilde{\phi}, \tag{10}$$

$$\dot{\tilde{\phi}} = -k\tilde{\phi} - k^2\tilde{\delta} + \dot{f}. \tag{11}$$

In view of the above error dynamics, we further have

$$\tilde{\delta}\dot{\tilde{\delta}} = -(\alpha - k)\tilde{\delta}^2 + \tilde{\delta}\tilde{\phi}, \tag{12}$$

$$\tilde{\phi}\dot{\tilde{\phi}} = -k\tilde{\phi}^2 - k^2\tilde{\phi}\tilde{\delta} + \dot{f}\tilde{\phi}. \tag{13}$$

The overall estimation process can be concluded as:

Theorem 1. *Consider the gimbal fault model in* (6) *with loss of effectiveness fault and additive bias fault. Applying the proposed local adaptive estimator consisting of a state estimation* (8) *and an auxiliary variable estimation* (9) *with the parameter constraints*

$$k - \alpha < 0, \tag{14}$$

$$k^4 + 2k^2 - (2\alpha + \epsilon)k + \alpha\epsilon + 1 < 0, \tag{15}$$

with ϵ being a small positive constant that is defined in the proof, the gimbal angle estimate error and fault estimate error will ultimately converge to small compact sets containing zero.

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\tilde{\delta}^2 + \frac{1}{2}\tilde{\phi}^2 \tag{16}$$

Taking time derivative of V and considering (12) and (13), we have

$$\begin{aligned}\dot{V} =& -(\alpha - k)\tilde{\delta}^2 - k\tilde{\phi}^2 - (k^2 - 1)\tilde{\delta}\tilde{\phi} + \dot{f}\tilde{\phi} \\ \le& -(\alpha - k)\tilde{\delta}^2 - \left(k - \frac{\epsilon}{2}\right)\tilde{\phi}^2 \\ & -(k^2 - 1)\tilde{\delta}\tilde{\phi} + \frac{1}{2\epsilon}\overline{f}^2,\end{aligned} \tag{17}$$

where the inequalities $\dot{f}\tilde{\phi} \le \frac{\epsilon}{2}\tilde{\phi}^2 + \frac{1}{2\epsilon}\overline{f}^2$ with ϵ being a small constant and $|\dot{f}| \le \overline{f}$, are used.

The foregoing inequality can be further written as

$$\dot{V} \le -\begin{bmatrix}\tilde{\delta} & \tilde{\phi}\end{bmatrix} \boldsymbol{P} \begin{bmatrix}\tilde{\delta} & \tilde{\phi}\end{bmatrix}^T + \gamma, \tag{18}$$

where $\boldsymbol{P} = \begin{bmatrix}\alpha - k & \frac{1}{2}(k^2 - 1) \\ * & k - \frac{\epsilon}{2}\end{bmatrix}$ and $\gamma = \frac{1}{2\epsilon}\overline{f}^2$.

When (14) and (15) are satisfied, it is clear that the matrix $\boldsymbol{P}$ is positive-definite. Consequently, we have $\dot{V} < -\kappa V + \gamma$ with $\kappa = 2\lambda_{\min}(\boldsymbol{P})$. Moreover, it is clear that $\dot{V} < 0$ when

$$|\tilde{\delta}(t)| > \sqrt{\frac{\gamma}{\lambda_{\min}(\boldsymbol{P})}} \ \texttt{or} \ |\tilde{\phi}(t)| > \sqrt{\frac{\gamma}{\lambda_{\min}(\boldsymbol{P})}}. \tag{19}$$

Therefore, $\tilde{\delta}$ and $\tilde{\phi}$ exponentially converge to compact sets with rates greater than $e^{-\kappa t}$.

$$\mathbb{S}_{\tilde{\delta}} = \left\{\tilde{\delta}\middle| |\tilde{\delta}| \le \sqrt{\frac{\gamma}{\lambda_{\min}(\boldsymbol{P})}}\right\}, \tag{20}$$

$$\mathbb{S}_{\tilde{\phi}} = \left\{\tilde{\phi}\middle| |\tilde{\phi}| \le \sqrt{\frac{\gamma}{\lambda_{\min}(\boldsymbol{P})}}\right\}. \tag{21}$$

In addition, since $\tilde{f} = \tilde{\phi} + k\tilde{\delta}$, the fault estimation error converges also converges to a compact set.

$$\mathbb{S}_{\tilde{f}} = \left\{\tilde{f}\middle| |\tilde{f}| \le (k+1)\sqrt{\frac{\gamma}{\lambda_{\min}(\boldsymbol{P})}}\right\}. \tag{22}$$

This completes the proof. □

IV. Fault-Tolerant Steering Logic

In this section, the PD feedback controller is adopted to achieve attitude tracking. We will proceed to develop a fault-tolerant SGCMG steering logic to allocate the torque calculated from attitude controller to each SGCMG while compensating effects caused by gimbal faults.

A. Spacecraft Attitude Dynamics

The kinematics and dynamics for attitude motion of a rigid spacecraft can be expressed by the ([21]):

$$\begin{cases} \boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\tau} + \boldsymbol{d} \\ \dot{\boldsymbol{q}} = \frac{1}{2}(\boldsymbol{q}^{\times} + q_0\boldsymbol{I}_3)\boldsymbol{\omega} \\ \dot{q}_0 = -\frac{1}{2}\boldsymbol{q}^T\boldsymbol{\omega}, \end{cases} \tag{23}$$

where $\boldsymbol{J} \in \mathbb{R}^{3\times 3}$ denotes the positive definite inertia matrix of the spacecraft, $\boldsymbol{\omega} \in \mathbb{R}^3$ is the inertial angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, $\boldsymbol{Q} = \begin{bmatrix}q_1 & q_2 & q_3 & q_0\end{bmatrix}^T = \begin{bmatrix}\boldsymbol{q}^T & q_0\end{bmatrix}^T \in \mathbb{R}^4$ denotes the unit quaternion and satisfies the constraint $\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1$, $\boldsymbol{I}_3 \in \mathbb{R}^{3\times 3}$ denotes a 3-by-3 identity matrix, $\boldsymbol{\tau} \in \mathbb{R}^3$ denotes the internal control torque produced by N identical SGCMGs, $\boldsymbol{d}$ is the bounded external disturbance, and the notation $\boldsymbol{x}^{\times}$ for a vector $\boldsymbol{x} = \begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}^T$ is used to represent the skew-symmetric cross-product matrix given by:

$$\boldsymbol{x}^{\times} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

To address the attitude tracking problem, the desired attitude and the desired angular velocity of the spacecraft are denoted by unit quaternion $\boldsymbol{Q}_d = \begin{bmatrix}\boldsymbol{q}_d^T & q_{d0}\end{bmatrix}^T$ and $\boldsymbol{\omega}_d$. The attitude tracking error $\boldsymbol{Q}_e = \begin{bmatrix}\boldsymbol{q}_e^T & q_{e0}\end{bmatrix}^T$ is defined as the relative orientation between attitude $\boldsymbol{Q}$ and target attitude $\boldsymbol{Q}_d$ and is computed as $\boldsymbol{Q}_e = \boldsymbol{Q}_d^{-1} \otimes \boldsymbol{Q}$, where $\boldsymbol{Q}_d^{-1}$ is the inverse or conjugate of the desired quaternion, and "$\otimes$" denotes the quaternion multiplication operator of two unit quaternion $\boldsymbol{Q}_i = \begin{bmatrix}\boldsymbol{q}_i^T & q_{i0}\end{bmatrix}^T$ and $\boldsymbol{Q}_j = \begin{bmatrix}\boldsymbol{q}_j^T & q_{j0}\end{bmatrix}^T$ and is defined as follows:

$$\boldsymbol{Q}_i \otimes \boldsymbol{Q}_j = \begin{bmatrix} q_{i0}\boldsymbol{q}_j + q_{j0}\boldsymbol{q}_i + \boldsymbol{q}_i^\times \boldsymbol{q}_j \\ q_{i0}q_{j0} - \boldsymbol{q}_i^T \boldsymbol{q}_j \end{bmatrix}. \tag{24}$$

The angular velocity error $\boldsymbol{\omega}_e$ is given by $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{C}\boldsymbol{\omega}_d$, where $\boldsymbol{C}$ is the rotation matrix, which is defined as $\boldsymbol{C} = (q_{e0}^2 - \boldsymbol{q}_e^T\boldsymbol{q}_e)\boldsymbol{I}_3 + 2\boldsymbol{q}_e\boldsymbol{q}_e^T - 2q_{e0}\boldsymbol{q}_e^\times$. Consequently, based on the attitude dynamics in (23), the attitude tracking error system can be described as

$$\begin{cases} \boldsymbol{J}\dot{\boldsymbol{\omega}}_e = -(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times \boldsymbol{J}(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) \\ \qquad\quad + \boldsymbol{J}(\boldsymbol{\omega}_e^\times \boldsymbol{C}\boldsymbol{\omega}_d - \boldsymbol{C}\dot{\boldsymbol{\omega}}_d) + \boldsymbol{\tau}_a + \boldsymbol{d} \\ \dot{\boldsymbol{q}}_e = \frac{1}{2}(\boldsymbol{q}_e^\times + q_{e0}\boldsymbol{I}_3)\boldsymbol{\omega}_e \\ \dot{q}_{e0} = -\frac{1}{2}\boldsymbol{q}_e^T\boldsymbol{\omega}_e. \end{cases} \tag{25}$$

B. SGCMG Steering Logic Design

The steering law is to map the commanded control torque $\boldsymbol{u}$ to the gimbal rate $\dot{\boldsymbol{\delta}}$. For a given control torque command $\boldsymbol{u}$, the internal control torque generated by N SGCMG should satisfy

$$\boldsymbol{\tau} = -h_0\boldsymbol{A}\dot{\boldsymbol{\delta}} - \boldsymbol{\omega}^\times \boldsymbol{h} = \boldsymbol{u}, \tag{26}$$

where h_0 is the magnitude of nominal angular momentum, $\boldsymbol{A}$ is the the Jacobian matrix of the derivative of $\boldsymbol{h}$, $\boldsymbol{h}$ is the angular momentum produced by SGCMG cluster, and $\dot{\boldsymbol{\delta}} = [\dot{\delta}_1, \ldots, \dot{\delta}_N]^T \in \mathbb{R}^N$ is the actual gimbal rate vector.

Considering SGCMG gimbal fault modeled in (6), the actual gimbal rate output $\dot{\boldsymbol{\delta}}$ and the gimbal rate command $\dot{\boldsymbol{\delta}}_c = [\dot{\delta}_{c,1}, \ldots, \dot{\delta}_{c,N}]^T$ of N SGCMGs have the following relationship

$$\dot{\boldsymbol{\delta}} = \dot{\boldsymbol{\delta}}_c + \boldsymbol{f}. \tag{27}$$

To compensate the total SGCMG fault effects, the estimated fault information $\hat{\boldsymbol{f}} = [\hat{f}_1, \ldots, \hat{f}_N]^T$ from previous section is used to replace the actual fault $\boldsymbol{f} = [f_1, \ldots, f_N]^T$ in steering logic design. Then, substituting (27) into (26), the commanded gimbal rate $\dot{\boldsymbol{\delta}}_c$ of N SGCMGs in the presence of faults is chosen such that

$$-h_0\boldsymbol{A}(\dot{\boldsymbol{\delta}}_c + \hat{\boldsymbol{f}}) - \boldsymbol{\omega}^\times \boldsymbol{h} = \boldsymbol{u}. \tag{28}$$

To obtain the commanded gimbal rate $\dot{\boldsymbol{\delta}}_c$ while coping with SGCMG cluster singularity (rank($\boldsymbol{A}$) < 3 at some specific gimbal angles) and SGMCG faults, the following fault-tolerant general singular robust (GSR) steering logic [2] is proposed:

$$\dot{\boldsymbol{\delta}} = -\frac{1}{h_0}\boldsymbol{A}^\# \left(\boldsymbol{u} + \boldsymbol{\omega}^\times \boldsymbol{h} + h_0\boldsymbol{A}\hat{\boldsymbol{f}}\right), \tag{29}$$

where $\boldsymbol{A}^\# = \boldsymbol{A}^T\left[\boldsymbol{A}\boldsymbol{A}^T + \lambda\boldsymbol{E}\right]^{-1}$ with $\boldsymbol{A}$ being the Jacobian matrix, $\lambda = 0.01\exp\left[-10\det\left(\mathbf{A}\mathbf{A}^T\right)\right]$, and the matrix $\boldsymbol{E}$ is expressed as:

$$\mathbf{E} = \begin{bmatrix} 1 & \varepsilon_3 & \varepsilon_2 \\ \varepsilon_3 & 1 & \varepsilon_1 \\ \varepsilon_2 & \varepsilon_1 & 1 \end{bmatrix} > 0$$

with $\varepsilon_i = 0.01\sin(0.5\pi t + \phi_i)$, $\phi_1 = 0$, $\phi_2 = \pi/2$ and $\phi_3 = \pi$.

V. Numerical Simulation

To verify the proposed local fault estimators and fault-tolerant steering logic, we consider the three-axis attitude control problem of a SGCMG-actuated spacecraft subject to time-varying SGCMG gimbal faults.

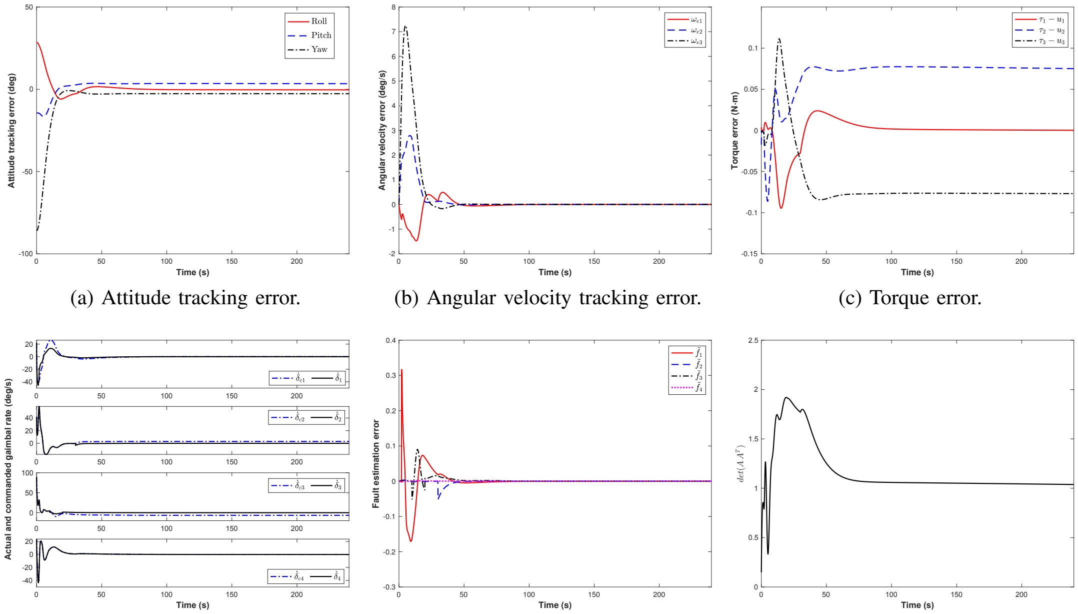
The spacecraft used for the simulation has the inertia $\boldsymbol{J} = \begin{bmatrix}10 & 1.2 & 0.5; 1.2 & 19 & 1.5; 0.5 & 1.5 & 25\end{bmatrix}$ kg·m^2. The environmental disturbance model is described by $\boldsymbol{d}(t) = \begin{bmatrix}-0.005\sin(t) & 0.005\sin(t) & -0.005\sin(t)\end{bmatrix}^T$ N·m. The spacecraft is required to perform a three-axis attitude maneuver in the space mission. The initial attitude of the spacecraft is assumed to be $\boldsymbol{Q}(0) = \begin{bmatrix}-0.5 & 0.3 & -0.4 & 0.7071\end{bmatrix}^T$, while the initial angular velocity is $\boldsymbol{\omega}(0) = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$ deg/s. Throughout the simulation, we consider a rest-to-rest attitude maneuver, in which the target attitude is $\boldsymbol{Q}(d) = \begin{bmatrix}0 & 0.8660 & 0 & 0.5\end{bmatrix}^T$. Four identical SGCMGs in a regular pyramid configuration with skew angle being $\beta = 54.74$ deg are used as actuators to reorient the spacecraft. The magnitude of nominal angular momentum of each SGCMG is 1 N·m·s. The Jacobian matrix of the angular momentum produced by the SGCMG cluster is given by

$$\boldsymbol{A} = \begin{bmatrix} -c\beta\cos\delta_1 & \sin\delta_2 & c\beta\cos\delta_3 & -\sin\delta_4 \\ -\sin\delta_1 & -c\beta\cos\delta_2 & \sin\delta_3 & c\beta\cos\delta_4 \\ s\beta\cos\delta_1 & s\beta\cos\delta_2 & s\beta\cos\delta_3 & s\beta\cos\delta_4 \end{bmatrix},$$

where $c\beta \equiv \cos\beta$, $s\beta \equiv \sin\beta$.

The fault scenario of the four SGCMGs is that the SGCMG #1 can only supply 50% of the commanded gimbal rate after $t = 2$ s, the SGCMG #2 experiences additive offset fault at $t = 30$ s with the size of $\dot{\delta}_{o_2} = -3$ deg/s, the SGCMG #3 is assumed to suffer from partial loss of effectiveness fault at $t = 10$ s with $\eta_3^g = 0.3$ and additive offset fault at $t = 20$ s with $\dot{\delta}_{o_3} = 2$ deg/s, and the SGCMG #4 is fault-free throughout the simulation. If the proposed fault-tolerant attitude control system can handle these severe SGCMG faults, it of course can deal with the less-severe SGCMG faults or fault-free situation. The initial gimbal angles of four SGCMGs are set to be $\boldsymbol{\delta}(0) = \begin{bmatrix}15 & 105 & 195 & -75\end{bmatrix}^T$ deg, which is near the hyperbolic internal singularities. To achieve three-axis attitude traking, a PD feedback controller [22] is used to generate the high-level torque command:

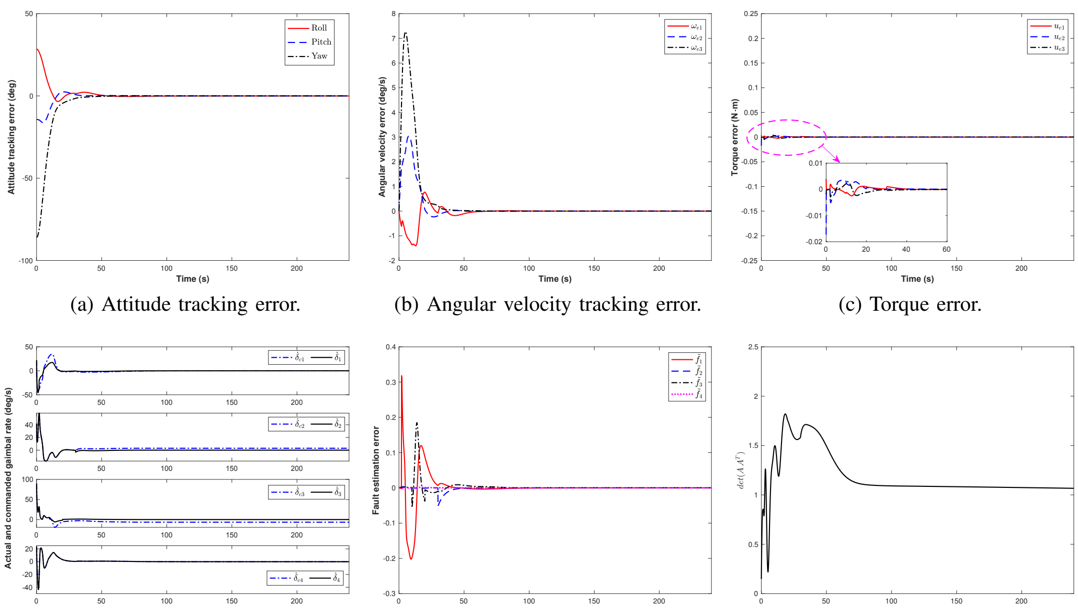
$$\boldsymbol{u} = -k_p\boldsymbol{J}\boldsymbol{q}_e - k_d\boldsymbol{J}\boldsymbol{\omega}_e + \boldsymbol{\omega}^\times\boldsymbol{J}\boldsymbol{\omega} - \boldsymbol{J}\left(\boldsymbol{\omega}_e^\times\boldsymbol{C}\boldsymbol{\omega}_d - \boldsymbol{C}\dot{\boldsymbol{\omega}}_d\right), \tag{30}$$



(a) Attitude tracking error. (b) Angular velocity tracking error. (c) Torque error.

(d) Actual and commanded gimbal rate. (e) Fault estimation error. (f) Singularity measure

Fig. 2: Attitude control performance under the local adaptive fault estimator and existing GSR steering logic without fault compensation.



(a) Attitude tracking error. (b) Angular velocity tracking error. (c) Torque error.

(d) Actual and commanded gimbal rate. (e) Fault estimation error. (f) Singularity measure

Fig. 3: Attitude control performance under the proposed control scheme using local adaptive fault estimator and fault-tolerant GSR steering logic.

where k_p and k_d are two control gains selected as $k_p = 0.1422$ and $k_d = 0.5333$.

For the purpose of comparison, the attitude control system consisting of PD controller (30), local fault estimator and conventional GSR steering logic without fault compensation is also implemented. Since the conventional GSR steering logic does not compensate the fault effects in steering logic design, obvious torque error between the commanded torque from attitude controller and the output torque generated by the SGCMG cluster is observed, as shown in 2c. This torque error further leads to a large steady-state attitude tracking error in attitude control, which can be observed in Fig. 2a. Although the SGCMG faults are not compensated in the conventional GSR steering logic, it is clear from Fig. 2e that the local fault estimator still estimates the fault of each SGCMG accurately and quickly.

To demonstrate the effectiveness of the proposed attitude control system, the simulation results with the PD controller (30), local fault estimator and fault-tolerant steering logic (29) are shown in Fig. 3. It is observed from Figs. 3a and 3b that the attitude and angular velocity tracking errors converge to a small neighborhood of zero under the proposed fault-tolerant attitude control scheme within 100 s, which outperforms the performance of attitude tracking errors shown in Fig. 2a where SGCMG faults are not taken into account in steering logic design. Comparing the torque errors of two steering logics, i.e., Fig. 3c and Fig. 2c, it is clear that the proposed fault-tolerant GSR steering logic reduces the torque error significantly, which verifies the effectiveness of our approach. Fig. 3d shows the actual and commanded gimbal fault, where fault effects, such as effectiveness decrease of CMG$\#1$ and additive offsets of CMG$\#2$ and CMG$\#3$, are observed clearly. For the CMG$\#4$, the actual gimbal rate follows the commanded value well since it is fault free. Fig. 3e shows the fault estimation errors of local estimators, which indicate that each lumped control effect caused by gimbal fault are estimated accurately and the estimation errors converge to zero. The singularity measure $\det(\boldsymbol{A}\boldsymbol{A}^T)$ is shown in Fig. 3f, from which it is observed that the SGMCG cluster does not fall into hyperbolic internal singularities although the initial gimbal angle is close to that singularity.

VI. Conclusions

In this paper, the fault modeling, fault estimation and fault-tolerant strategy are proposed to handle the SGCMG gimbal fault. First, the SGCMG is considered as a combination of two independent EM-VSD systems, which describe the rotor speed control loop and gimbal rate control loop, respectively. The potential faults of the SGCMGs in each EM-VSD loop are analyzed and modeled by an effectiveness matrix and an additive offset. Then the fault model of the SGCMG is described in a cascade multiplicative form by combining two EM-VSD fault models. Based on this fault model, local fault estimators with respect to each SGCMG are developed to estimate the total fault effect instead of each individual fault. It is shown that the proposed local fault estimator can exponentially converge the fault estimation error with satisfactory accuracy. Moreover, incorporating the estimated fault information, a fault-tolerant steering logic is proposed to accommodate SGCMG gimbal fault while generating a proper gimbal rate command. Finally, the effectiveness of the proposed SGCMG fault estimation approach and fault-tolerant steering logic is demonstrated by numerical simulation of a rigid spacecraft subject to time-varying gimbal faults.

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