Fault-Tolerant Attitude Control of Spacecraft via Explicit Model Prediction Method

Xin Cao, Deren Gong, Qiang Shen, and Shufan Wu

Abstract In this paper, the problem of spacecraft attitude control with soft faults of actuators is investigated. A robust fault-tolerant controller is proposed in spacecraft with the explicit model predictive control. Firstly, Fault model of the actuator is established with the method based on explicit model prediction. Then, considering the model uncertainty and the system state disturbance, the control problem based on the spacecraft actuator failure state was transformed into the multi-parametric quadratic programs (MPQP) under the constraints. Finally, a recursive process of combining and replacing solutions is given to extract the required explicit control laws. By designing the terminal cost function and constraint set appropriately, it is proved that the MPC controller is robust to the constraints applied in the closed loop of the uncertain system and the input to the stability of the origin state.

Keywords Passive fault-tolerant control · Model predictive control · Multiparameter programming · Dynamic programming · Fault-tolerant control in spacecraft

1 Introduction

In the spacecraft attitude control, failure of an actuator may cause irreparable damage and inevitably appear disturbance and uncertainty. Therefore, the design of the control system should not only be robust to disturbances and uncertainties, but also be fault-tolerant. Control systems that maintain overall stability and acceptable levels under fault conditions are called fault-tolerant control systems (FTC).

With the continuous development of fault diagnosis technology, the corresponding fault-tolerant control methods are also gradually developed, especially the method

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Z. Jing and D. Strelets (eds.), *Proceedings of the International Conference on Aerospace System Science and Engineering 2021*, Lecture Notes in Electrical Engineering 849,
https://doi.org/10.1007/978-981-16-8154-7_12

based on analytical model. This method is often divided into two categories, namely, passive fault-tolerant control (PFTC) method and active fault-tolerant control method (AFTC). AFTC method is to adjust the parameters of the controller according to the fault (after the fault occurs) to compensate for the fault, and the structure needs to be changed if necessary. Obviously, this method requires a designed control algorithm. However, this method can improve the performance of the controlled system. Generally speaking, most active fault-tolerant controls require fault diagnosis (FD) subsystems or modules to obtain fault-related information [1]. By contrast, PFTC method is based on the idea of robust control [2]. In related reports, passive fault-tolerant control is often divided into reliable stabilization, simultaneous stabilization, and integrity. Passive fault-tolerant control method is essentially a robust control technology, which is effective only for faults in a specific range, and generally speaking, it is more conservative [3].

At present, different FTC methods have been proposed, but Model Predictive Control (MPC) has not been used to solve the control problem of spacecraft actuators under passive FTC. The main problem of MPC is the large amount of calculation online, which limits its wide application. Considering the limited power supply, data storage, and computing resources of spacecraft, an explicit predictive control (EMPC) is proposed to solve this problem [4]. In this method, the optimal solution of the closed-loop system is obtained offline. The original MPC problem is modified by dynamic programming and multi-parameter programming, and it is transformed into a multi-stage optimization problem [5]. However, for the design of spacecraft actuators, there is no robust fault-tolerant controller for disturbance and uncertainty.

In this paper, the factors such as actuator failure, disturbance, and model uncertainty are combined into the predictive control problem of spacecraft linear systems, robust fault-tolerant explicit control laws under state and input constraints are derived. On the basis of the work of Kourama, the algorithm adopts the method of combining dynamic programming with multi-parameter programming [6]. By using the idea of constraint rearrangement, the control problem based on the failure state of spacecraft actuator is transformed into a constrained multi-parameter optimization problem. Finally, the recursive process of combinatorial and substitution solutions is given to obtain the required explicit control law.

2 Problem Formation

Considering the following satellite model assumptions:

- All state variables in the model are based on the satellite body-fixed frame O_b.
- The origin of O_b coincides with the center of gravity of the satellite, the inertial matrix of a satellite is $\mathbf{I} = \mathrm{diag}\{i_{11}, i_{22}, i_{33}\}$.
- The satellite's reaction wheel generates internal control torque around the satellite's main axis, the axial inertia of the momentum wheel is $\mathbf{I}_s$, The axis of rotation in the ontology coordinate system is $\mathrm{C} = [0, 1, 0]^T$.

- Satellite external control moment $\sum \boldsymbol{\tau}_{\text{control}} \triangleq \boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^{\text{T}}$, provided by thrusters, can act directly on the angular velocity of the satellite's main axis, the only interfering moment to be considered is the gravity gradient moment.

2.1 Satellite Attitude Model

A preliminary and relatively complex satellite model can be described as

$$\begin{aligned} \dot{\omega}_{ib}^{b} &= \mathbf{J}^{-1}\left[-\mathbf{S}\left(\omega_{ib}^{b}\right)\left(\mathbf{I}\omega_{ib}^{b} + \mathrm{C}\mathbf{I}_s\omega_s\right) + \tau_e - \mathbf{J}^{-1}\mathrm{C}\tau_a\right] \\ \dot{\omega}_s &= -\mathrm{C}^{\mathrm{T}}\mathbf{J}^{-1}\left[-\mathbf{S}\left(\omega_{ib}^{b}\right)\left(\mathbf{I}\omega_{ib}^{b} + \mathrm{C}\mathbf{I}_s\omega_s\right) + \tau_e\right] + \left[\mathrm{C}^{\mathrm{T}}\mathbf{J}^{-1}\mathrm{C} + \mathbf{I}_s^{-1}\right]\tau_a \\ \dot{\eta} &= -\frac{1}{2}\epsilon^{\mathrm{T}}\omega_{ob}^{b} \\ \dot{\epsilon} &= \frac{1}{2}[\eta\mathbf{I} + \mathbf{S}(\epsilon)]\omega_{ob}^{b} \end{aligned} \tag{1}$$

where $\mathbf{J}$ is the satellite's overall inertia matrix, $\mathbf{I}$ is the inertial matrix of the satellite, $\mathbf{I}_s$ is the axial inertia matrix of reaction wheel, C is the matrix for the reaction wheel, $\boldsymbol{\omega}_s$ is the angular velocity of the reaction wheel, $\boldsymbol{\tau}_a$ is the reaction wheel control torque.

$$\boldsymbol{\tau}_e = \sum \boldsymbol{\tau}_{control} + \sum \boldsymbol{\tau}_{disturbance}, \ \sum \boldsymbol{\tau}_{control} \triangleq \boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^{\mathrm{T}} \tag{2}$$

$$\mathbf{S}(\boldsymbol{\omega}_{ib}^{b}) \triangleq \omega^{\times} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}, \ \boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}. \tag{3}$$

2.2 Satellite Attitude Model

Model in (1) must be differentiated relative to the total state vector, the total state vector is currently selected as $x = [\omega_1, \omega_2, \omega_3, \omega_s, \eta, \varepsilon_1, \varepsilon_2, \varepsilon_3]^T$. In this way, the nonlinear control law $\mathbf{f}(\mathbf{x}, \mathbf{u}) = [\dot{\omega}_{ob}^{b}, \dot{\omega}_s, \dot{\eta}, \dot{\epsilon}]^{\mathrm{T}} \triangleq [f_1, \ldots, f_8]^{\mathrm{T}}$. The linearized system can be expressed as follows:

$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} = C\mathbf{x} + D\mathbf{u} \end{cases}. \tag{4}$$

Considering the observability and controllability of linear systems, select equilibrium point $\mathbf{x}^p = [0, 0, 0, 0, 1, 0, 0, 0]^{\mathrm{T}}$, $\mathbf{u}^p = [0, 0, 0, 0]$, the solution formula of the state equation is used to ensure that the continuous state equation and the discretized state equation have the same solution at the sampling time. Transform the state-space model of discrete system into the following form:

$$\begin{cases} \mathbf{x}((k+1)T) = G(T)\mathbf{x}(kT) + H(T)\mathbf{u}(kT) \\ \mathbf{y}(kT) = C(T)\mathbf{x}(kT) + D(T)\mathbf{u}(kT) \end{cases} \tag{5}$$

Decoupling this system into a six-degree-of-freedom state system, the matrix in (6) is as follows:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & (1-k_x)\omega_0 & 0 & -8k_x\omega_0^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-6k_y i_{22}\omega_0^2}{\kappa} \\ (k_z-1)\omega_0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{i_{11}} & 0 & 0 \\ 0 & \frac{1}{\kappa} & 0 \\ 0 & 0 & \frac{1}{i_{33}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

where

$$k_x = \frac{i_{22}-i_{33}}{i_{11}},\ k_y = \frac{i_{11}-i_{33}}{i_{22}},\ k_z = \frac{i_{22}-i_{11}}{i_{33}},\ \kappa = i_{22} - i_s.$$

3 Fault Model Prediction Description

3.1 Linear Discrete System Under Spacecraft Failure

The state equation of linear discrete spacecraft is as follows:

$$x_{k+1} = f(x_k, u_k) = Ax_k + B\Omega_k u_k + Wd_k, \tag{7}$$

where $x_k \in R^n, u_k \in R^m, d_k \in R^s$ are state, input, and disturbance vector respectively, subject to constraints $\mathcal{X} = \{x \in R^n Gx \leq \mu\}$ and $u = \{u \in R^m Hu \leq \gamma\}$ are convex polyhedron, $G \in R^{n_M}$, $\mu \in R^{n_M}$, $H \in R^{m_N \times m}$, $\gamma \in R^{m_N}$; additionally, unknown disturbances d_k are restricted to $d_i^L \le d_{k,i} \le d_i^U (i = 1, \ldots, r)$, system coefficient matrix A, B are uncertain matrices; unknown boundary matrix $\Delta A \Delta B$ are defined as follows:

$$\Delta A \in \mathrm{A} = \left\{ \Delta A \in \mathbb{R}^{n\times n} | -\varepsilon_a |A_0| \le \Delta A \le \varepsilon_a |A_0 \right\} \tag{8}$$

$$\Delta B \in \mathrm{B} = \left\{ \Delta B \in \mathbb{R}^{n\times n} | -\varepsilon_b |B_0| \le \Delta B \le \varepsilon_b |B_0 \right\}, \tag{9}$$

where A_0 B_0 are nominal matrices, use $|X|$ represent $\{|X_{ij}|\}$, $X = \{X_{ij}\}$, $u_{k,i}$ stands for the ith actuator of time k and $u_{k,i}^F$ denotes its defective form written as

$$\varepsilon_{k,i} \in R = \left\{ \varepsilon_{k,i} \in \mathbb{R} | 0 \le \varepsilon_{k,i} \le \varepsilon_i^U \le 1 \right\}, \tag{10}$$

whereas $\varepsilon_{k,i}$ and ε_i^U indicate the corresponding failure percentage at time instant k and its upper bound, respectively. Therefore, $\varepsilon_{k,i} = 0$ indicates the state of health while $\varepsilon_{k,i} = 1$ indicates the correspondence of input is completely invalid. Now let's define

$\Omega_k \triangleq I_m - \varepsilon_k$, $\varepsilon_k \triangleq \mathrm{diag}\{\varepsilon_{k,1}, \ldots, \varepsilon_{k,m}\}$, I_m denote the unit matrix of order m, $u_k^F = \Omega_k u_k$, so the MPC problem under constraints is defined as[7]:

$$\min_{U^t} J = \sum_{k=0}^{N-1} \left[\bar{x}_{k|t}^{\mathrm{T}} Q \bar{x}_{kt} + u_{k|t}^{\mathrm{T}} R u_{k|t} \right] + \bar{x}_{Nt}^{\mathrm{T}} P \bar{x}_{N|t}, \tag{11}$$

where

$$\bar{x}_{k+1|t} = A_0 \bar{x}_{k|t} + B_0 u_{k|t}, k = 0, 1, 2 \ldots, N-1 \tag{12}$$

$$x_{k+1|t} = A x_{k|t} + B \Omega_k u_{k|t} + W d_{k|t}, k = 0, 1, 2, \ldots, N-1 \tag{13}$$

$$u_{k|t} \in \mathcal{U} = \left\{ u \in R^m, Hu \le \gamma \right\}, k = 0, 1, 2 \ldots, N-1 \tag{14}$$

$$x_{N|t} \in X_f = \left\{ x \in R^n, G_f x \le \mu_f \right\} \tag{15}$$

and $\forall \Delta A_{0|t}, \ldots, \Delta A_{k-1|t}$ satisfies (5), $\forall \Delta B_{0|t}, \ldots, \Delta B_{k-1|t}$ satisfies (6)$\forall \alpha_{0t}, \ldots, \alpha_{k-1|t}$ satisfies (7).

A control sequence $\mathbf{U}^t = \{u_{0|t}, \ldots, u_{N-1|t}\}$ is a robust solution for the explicit/multi-parametric MPC problem, $x_{0|t} = x_t$, the matrices Q, P are positive semi-definite, R is positive definite and X_f contains the origin in its interior. N is the prediction length. At each sampling time, the objective of this work is to obtain a control sequence U and in extension the optimal control variables u_t for the optimization as functions of the state variables x_t (7), it is obvious that deriving the control variables utas explicit functions of the states x_t is not possible with the current nominal mp-MPC methods which meets the constraints of all allowable values of disturbance, uncertainty, and actuator fault. In order to be applied to multi-parameter programming, it is assumed that the objective function only punishes the behavior of nominal systems (such as Kourama 2013). This closed-form solution of the problem is named as a robust fault-tolerant explicit control law.

3.2 Model Prediction Dynamic Programming

In this step, based on the ideas of multi-stage decomposition, problem (11)–(13) with N decision variables is expressed as a multi-stage optimization problem where the time instant t represents each of the stages of the problem with N optimization problems and one decision variable, that is

$$\min_{U_t} J = \sum_{k=0}^{N-1} \left[\bar{x}_{k|t}^{\mathrm{T}} Q \bar{x}_{k|t} + u_{k|t}^{\mathrm{T}} R u_{k|t}\right] + \bar{x}_{N|t}^{\mathrm{T}} P \bar{x}_{N|t} \tag{16}$$

where

$$\begin{aligned} &\bar{x}_{k+1} = A_0 \bar{x}_k + B_0 u_k,\, k = i, \ldots, N-1 \\ &x_i \in \chi,\, u_i \in \mathcal{U},\, x_{i+1} \in X_{i+1} = \left\{x \in R^n,\, G_{i+1} x \le \mu_{i+1}\right\}. \end{aligned} \tag{17}$$

The optimization problem (16) is solved stage wise, starting from $i = N-1$ and solving it repetitively backwards until $i = 0$, where effects of all uncertain parameters are captured as $x_{i+1} = Ax_i + B\Omega_i u_i$,where the only optimization variable is the control variable u_i at the current stage and only the state and input constraints at the current stage are considered [8].

3.3 Model Prediction Constraint Arrangement

The key step in the proposed method is to rearrange the constraint beat to ensure the robustness and fault tolerance of our controller, all constraints (17)–(20) can be rewritten as

$$\begin{aligned} &G_{i+1} A_0 x_i + G_{i+1} \Delta A_i x_i + G_{i+1} B_0 u_i - G_{i+1} B_0 \varepsilon_i u_i + \\ &G_{i+1} \Delta B_i u_i - G_{i+1} \Delta B_i \varepsilon_i u_i + G_{i+1} W d_i \le \mu_{i+1}, \end{aligned} \tag{18}$$

Considering the perturbation, uncertainty and worst case of actuator failure, the model constraint prediction is rearranged as follows:

$$\begin{aligned}&G_{i+1}A_0x_i + G_{i+1}B_0u_i\\ &\quad + \max(G_{i+1}\Delta A_i x_i - G_{i+1}B_0\varepsilon_i u_i + G_{i+1}\Delta B_i u_i\\ &\quad - G_{i+1}\Delta B_i\varepsilon_i u_i + G_{i+1}Wd_i) \le \mu_{i+1}\end{aligned} \tag{19}$$

$$-y_i \le x_i \le y_i \tag{20}$$

$$-v_i \le u_i \le v_i. \tag{21}$$

3.4 Multi-parametric Programming

The convexity of (16) can be used to derive a convex multi-parameter programming problem by considering the assumptions as follows [9]:

- Considering $\varphi_i = \left[u_i^T, y_i^T, v_i^T\right]^T$ as the optimization variable.
- Considering $\theta_i = \left[x_i^T, u_{i+1}^T, \ldots, u_{N-1}^T\right]^T$ as the optimization parameters.
- Considering objective function only penalizes the nominal system $\bar{x}_{k+1} = A_0\bar{x}_k + B_0u_k$, therefore $\bar{x}_{k+1} = A_0^{k-i+1}x_i + \sum_{j=i}^{k} A_0^{k-j}B_0u_j (k = i, \ldots, N-1)$.

The design of robust controller control law for spacecraft is transformed into the multi-parameter quadratic optimization problem as follows:

$$\min_{\varphi_i}\left\{\frac{1}{2}\varphi_i^{\mathrm{T}} H_i\varphi_i + \theta_i^{\mathrm{T}} F_i\varphi_i\right\} \tag{22}$$

$$A_{ci}\varphi_i \le b_{ci} + B_{ci}\theta_i, \tag{23}$$

where

$$H = R + \sum_{i=t}^{N-1} B_0^{\mathrm{T}} \left(A_0^{i-1-t}\right)^{\mathrm{T}} Q A_0^{i-1-t} B_0$$

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$$F = \left[\left(\sum_{t+1}^{N-1} 2\left(A_0^{i-t}\right)^{\mathrm{T}} Q A_0^{i-1-t} \right)^T \times \left(\sum_{i=t+2}^{N-1} 2B_0^T \left(A_0^{i-t-2}\right)^{\mathrm{T}} Q A_0^{i-1-t} B_0 \right) \right.$$
$$\left. \times \left(\sum_{i=t+3}^{N-1} 2B_0^T \left(A_0^{i-t-3}\right)^T Q A_0^{i-1-t} B_0 \right)^{\mathrm{T}} \cdots \left(B_0^T P A_0^{N-1-t} B_0\right) \right]$$

In addition, A_{ci},b_{ci},B_{ci} is simply obtained from constraints (18)–(20), By using the transformation $z_i \triangleq \varphi_i + H_i^{-1} F_i^{\mathrm{T}} \theta_i$, the problem is converted to the standard form of the multi-parameter quadratic programming problem, which is

$$\min_{z_i} \frac{1}{2} z_i^T H_i z_i \tag{24}$$

$$A_{ci} z_i \leq b_{ci} + S_i \theta_i, \tag{25}$$

where $S_i = A_{ci} H_i^{-1} F_i^{\mathrm{T}} + B_{ci}$, it should be a positive symmetric definite matrix, a problem solved by Yalmip in the multi-parameter toolbox, the solution in MPT takes the form of [10]:

$$\begin{aligned} u_i &= K_i^j \theta_i + k_i^j \\ if\ \theta_i \in \mathcal{CR}_i^j &= \left\{ \theta A_{\theta_i}^j \theta \leq b_{\theta_i}^j \right\}. \end{aligned} \tag{26}$$

At the end of the current stage, the t-robust controllability set for the feasibility constraint in (17) can be obtained either by performing set theoretic computations, or by taking the union that allowable constraint region of the next stage is calculated as follows:

$$\mathcal{X}_i = \bigcup_j c\mathcal{R}_i^j. \tag{27}$$

At the end of each phase, the solution of form (26) is obtained through this method, but the expected control law is only a function of x_i. These formulas (28)–(30) show how to extract the required explicit control law by combining the solution of previous and current stages which are obtained from the algorithm:

$$u_i = K^r_{x_i} x_i + K^r_{u_i} u_{pre} + k^r_i \ if \ A^r_{x_i} x_i + A^r_{u_i} u_{pre} \le b^r_{\theta_i} \tag{28}$$

$$u_{\text{pre}} = K^q_{\text{pre}} x_{i+1} + k^q_{\text{pre}} \ if \ A^q_{\text{pre}} x_{i+1} \le b^q_{\text{pre}}, \tag{29}$$

where $u_{\text{pre}} = \left[u^T_{i+1}, \ldots, u^T_{N-1}\right]^T$ and K^q_{pre} , k^q_{pre} A^q_{pre} , b^q_{pre} are obtained at the end of previous stage for the critical region, $\mathcal{CR}^q_{\text{pre}}$, rewrite the system using the nominal model $x_{i+1} = A_0 x_i + B_0 u_i$ in the following form:

$$\begin{bmatrix} I_m & -K^r_{u_i} \\ -K^q_{pre} B_0 & I_{m\times(N-i-1)} \end{bmatrix} \begin{bmatrix} u_i \\ u_{pre} \end{bmatrix} = \begin{bmatrix} k^r_i \\ k_{pre} \end{bmatrix} + \begin{bmatrix} K^r_{x_i} \\ K^q_{pre} A_0 \end{bmatrix} x_i . \tag{30}$$

It is possible to find two critical regions from $A^r_{x_i} x_i + A^r_{u_i} u_{pre} \le b^r_{\theta_i}$ and $A^q_{pre} A_0 x_i + A^q_{pre} B_0 u_i \le b^q_{pre}$, which is the critical region for the sequence from u_i to u_{N-1}, An empty intersection indicates that there is no feasible solution in the current $r - q$ combination. This procedure is required to being repeated at the end of each stage. In this way, the control problem based on the failure state of spacecraft actuator is transformed into a multi-parameter optimization problem under constraints.

4 Simulation

4.1 2U Cubic Satellite Parameters Initialized

Table 1 summarizes some of the main physical parameters. These values were used in the previous analysis and will be used in the simulation section of this paper [11], adopting the satellite attitude kinematics and dynamics model in the paper, the operating altitude of the micro-nano satellite is 17125 km, the moment of inertia of the satellite on three axes is $I = diag\{4.251, 4.330, 3.669\}[kg\, m^2]$, [12] the satellite's state variable is $x = [\omega_1, \omega_2, \omega_3, \eta, \alpha_1, \alpha_2, \alpha_3]^{\text{T}}$,$\omega_1, \omega_2, \omega_3$ are, respectively, satellite triaxial angular velocities, $\eta, \alpha_1, \alpha_2, \alpha_3$ are the corresponding quaternions, respectively. In order to facilitate tracking and control, the state quantity of the satellite is set as $x = [\omega_1, \omega_2, \omega_3, \alpha_1, \alpha_2, \alpha_3]^{\text{T}}$. The fixed sampling time is 0.1 s, and the state constraint is

Table 1 Observation from the state partition

Observe	Fixed manipulation	Value
ω_1, ε_1	$\omega_2, \varepsilon_2, \omega_3, \varepsilon_3$	$case1 = [0, 0, 0, 0]$
ω_2, ε_2	$\omega_2, \varepsilon_2, \omega_3, \varepsilon_3$	$case2 = [0, 0, 0, 0]$
ω_3, ε_3	$\omega_2, \varepsilon_2, \omega_3, \varepsilon_3$	$case3 = [0, 0, 0, 0]$

$$-[0.5; 0.5; 0.5; 0.5; 0.5; 0.5] <= X <= [0.5; 0.5; 0.5; 0.5; 0.5; 0.5];$$

The input constraint is:

$$-[0.484; 0.484; 0.039] <= U <= [0.484; 0.484; 0.039];$$

The state matrix is Q = diag(200,200,200,1,1,1);

The input matrix is R = diag(100,200,100);

The uncertainty of system model is $\sigma = 0.1$, upper limit of the fault $\alpha^U = 0.5$. The predicted step size is $N = 4$, and the system control partition obtained by offline calculation is shown in Table 1.

4.2 Simulation Results

In order to study the high performance of the designed controller, the behavior of the closed-loop system starting from any initial condition x_0 is simulated. As can be seen from Fig. 1, which illustrates the explicit region of the satellite rolling angular velocity and rolling angle, the explicit region of the satellite pitch velocity and pitch angle is depicted in Fig. 2. Figure 3 also reflects the explicit region of the yaw angular velocity and yaw angle.

Taking pitch angle and yaw angle as an example, their control law partitions are indicated in Fig. 4, corresponding to u_1, u_2, u_3 from top to bottom. In this way, different control law permutation and combination under different states can be obtained.

Figure 5 demonstrates the state partitions of all angular velocities when the roll angle, pitch angle, and yaw angle are special values of zero; Fig. 6 illustrates the

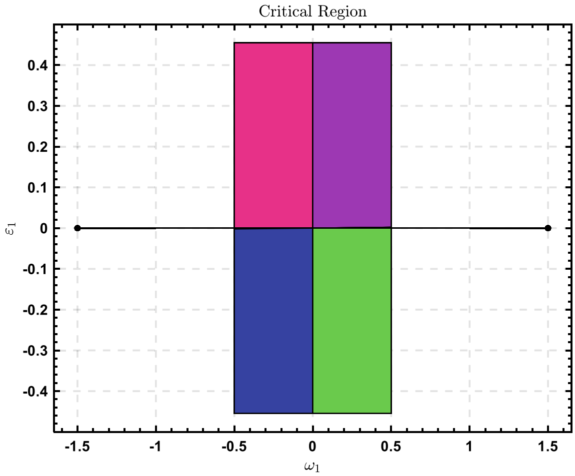


Fig. 1 Feasible critical region of the observation1

Fig. 2 Feasible critical region of the observation2

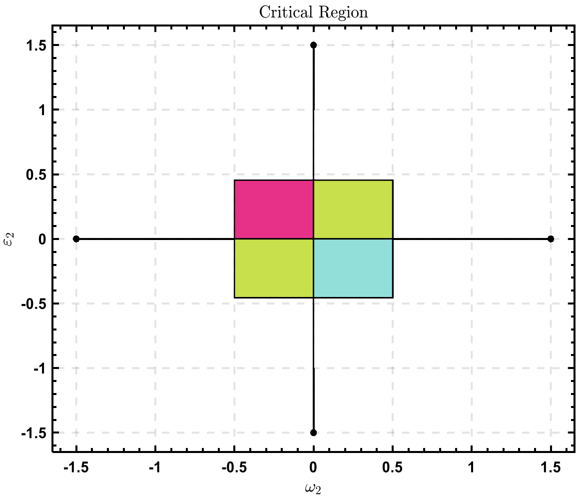
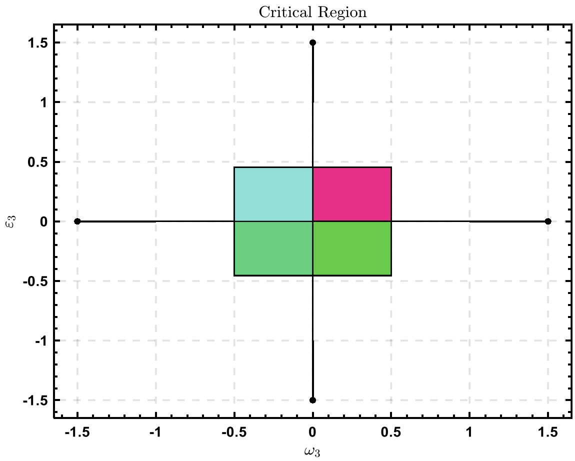


Fig. 3 Feasible critical region of the observation3



state partitions of all Euler parameters when the roll velocity, pitch velocity, and yaw velocity are special values of zero.

In order to study the high performance of the designed controller, the behavior of the closed-loop system starting from any initial condition $x_0 = [0.1; 0.2; 0.5; 0.3; 0.4; 0.1]$ is simulated. The system is affected by random allowable disturbance, uncertainty, and actuator failure. Figure 7 shows the curve of the three control laws changing with time, it can be seen that the control input basically reaches the balance at 4 s.

In Fig. 8 three curves of angular velocity over time are shown and Fig. 9 reflects curves of Euler quaternions over time, indicating that after a fault occurs, the system can quickly and timely search for an emergency plan through explicit predictive

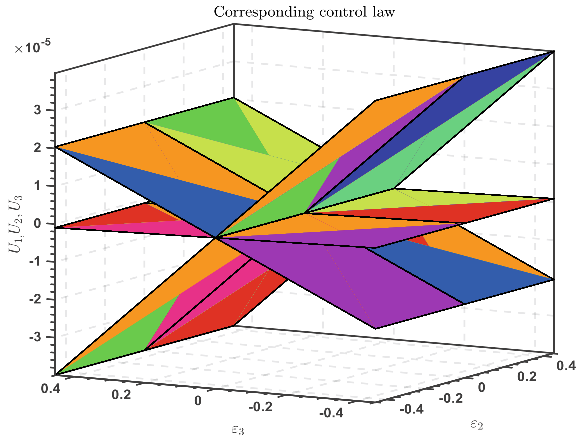


Fig. 4 Feasible region of the control law

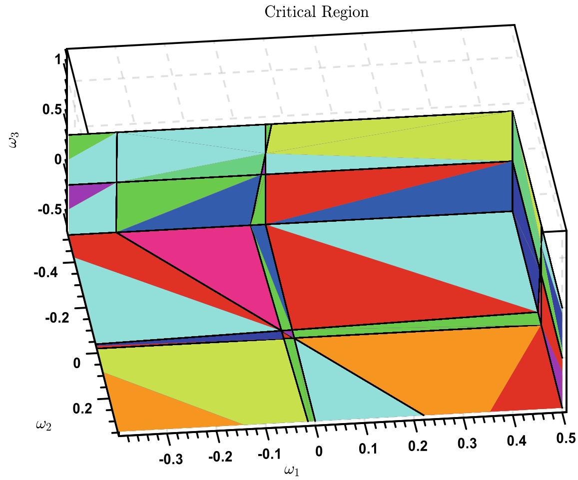


Fig. 5 Feasible region of all angular velocities

control, so as to stabilize the spacecraft attitude quickly. Especially, the transient response is faster and the steady-state accuracy is higher.

5 Conclusion

In this work, a new methodology was applied in micro-satellite to deal with a robust fault-tolerant explicit solution as an EMPC problem with objective quadratic function and linear state and input constraints. The micro-satellite system was exposed to unknown bounded disturbances, actuator failures as well as model uncertainties. A multi-stage optimization program was implemented by using the multi-parametric

Fig. 6 Feasible region of all Euler parameters

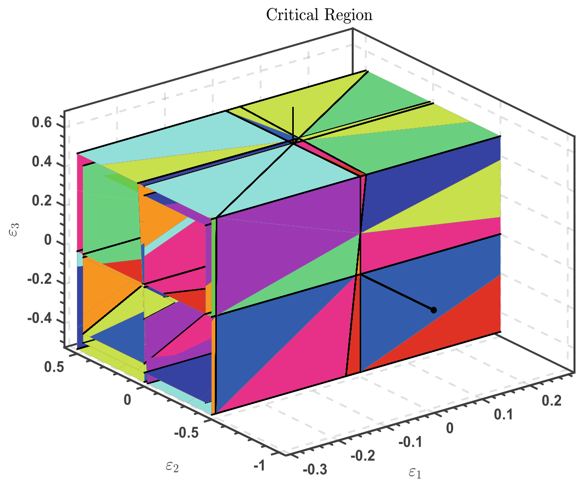
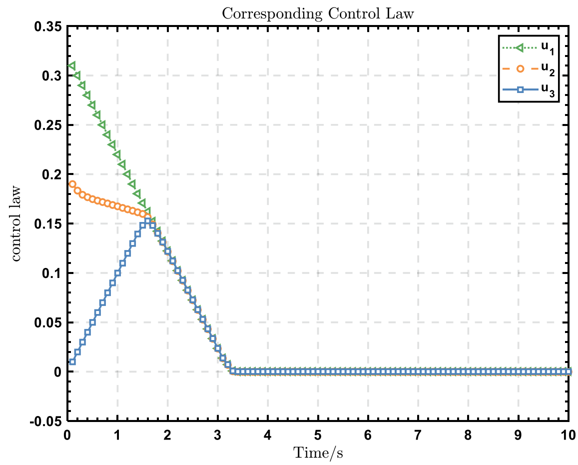


Fig. 7 Feasible region of control law



programming and dynamic programming approaches. Furthermore, an implemented constraint rearrangement formulation was imposed which ensures the immunity of the attitude system to the worst situation such as disturbances, uncertainties, and faults. Additionally, a recursive process was used to combine the stage-by-stage solutions and to extract the expected explicit control law. The numerical simulations show that the proposed controller is obviously successful in achieving high performance in quality as well as in the presence of parametric uncertainty, external disturbance, actuator failure, and control input constraints. At the same time, it provides an idea for cooperative control and distributed control of multi-spacecraft.

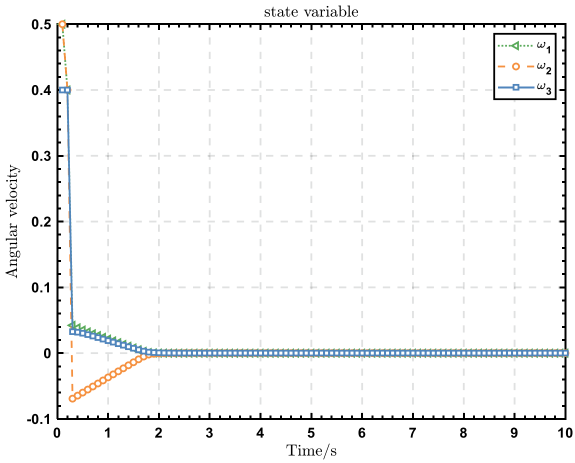


Fig. 8 Simulation results of the angular velocity

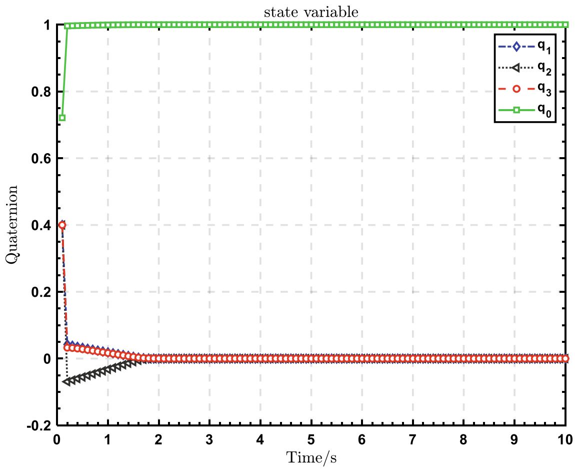


Fig. 9 Results of the Euler parameters with time

Acknowledgements This work is partially supported by National Natural Science Foundation of China, No. U20B2054, National Key R&D Program of China, No. 2020YFC2200800. We wish to thank Qiang Shen for his help with the initial implementation of the algorithm, and Deren Gong for suggesting improvements to the original manuscript.

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